TOTAL PAGES: 2

23MCU627

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

LINEAR ALGEBRA

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If A is an invertible n x n matrix, then for each b in R^n, the equation Ax = b has the unique solution _____
(i) $x = A^{-1}b$ (ii) $xb = A^{-1}$
(iii) $x = Ab$ (iv) $b = A^{-1}x$

2 Let V be a finite-dimensional vector space and let n = dim V. Then _____ subset of V which contains fewer than n vectors can span V.
(i) finite (ii) infinite
(iii) no (iv) only one

3 The null space of T is the set of all vectors α in V such that Tα = _____
(i) 1 (ii) ϕ (iii) ∞ (iv) 0

4 A finite-dimensional real inner product space is often called as a _____ space
(i) Euclidean (ii) unitary
(iii) finite (iv) infinite

5 Which of the following is an eigenvalue of A if and only if A is not invertible?
(i) 1 (ii) -1 (iii) ∞ (iv) 0

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If v_1 and v_2 are in R^n and H = Span $\{v_1; v_2\}$ then prove that H is a subspace of R^n.

OR

b Determine whether b is in the column space of A, when $A = \begin{pmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix}$.

7 a A non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in F the vector cα + β is again in W. Using logical proof, justify this statement.

OR

b The subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S. Justify this, using valid proof.

8 a Analyze that, if A is an m x n matrix with entries in the field F, then show that row rank (A) = column rank (A).

OR

b Considering V and W as vector spaces over the field F, prove that the set of all linear transformations from V into W, together with the addition and scalar multiplication, is also a vector space over the field F.

Cont...

23MCU627
Cont...

9 a If V is an inner product space, then for any vectors α, β in V and any scalar c, prove that (i) $|(\alpha \mid \beta)| \leq \|\alpha\| \, \|\beta\|$ (ii) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$

OR

b Prove that an orthogonal set of non-zero vectors is linearly independent.

10 a If n x n matrices A and B are similar, then prove that they have the same characteristic polynomial and hence the same eigenvalues (with the same multiplicities).

OR

b Obtain a formula for A^k, given that $A = PDP^{-1}$, where $A = \begin{pmatrix} 7 & 2 \\ -4 & 1 \end{pmatrix}$, $P = \begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Use the inverse of the matrix $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ to solve the system $3x_1 + 4x_2 = 3$, $5x_1 + 6x_2 = 7$.

OR

b If A and B are n x n matrices and E is an n x n elementary matrix, then prove that det EA = (det E)(det A) where $\det E = \begin{cases} 1 & \text{if E is a row replacement} \\ -1 & \text{if E is an interchange} \\ r & \text{if E is a scale by r} \end{cases}$

Also show that det AB = (det A)(detB).

12 a Prove that the field C of complex numbers is a vector space over the field R of real numbers.

OR

b If V is a finite-dimensional vector space, then prove that any two bases of V have the same (finite) number of elements. Analyze this statement.

13 a Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \ldots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\beta_1, \ldots, \beta_n$ be any vectors in W. Then prove that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j$, $j = 1, \ldots, n$.

OR

b Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then prove that the space L(V, W) is finite-dimensional and has dimension mn.

14 a Prove that every finite-dimensional inner product space has an orthonormal basis.

OR

b Let W be a finite-dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^{\perp}$ is the null space of E, and $V = W \oplus W^{\perp}$

15 a Analyze the long term behavior of the dynamical system defined by $x_{k+1} = Ax_i$ (k = 0, 1, 2, ...) with $x_0 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ and $A = \begin{pmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{pmatrix}$

OR

b Calculate the eigenvalues of $A = \begin{pmatrix} 0.5 & -0.6 \\ 0.75 & 1.1 \end{pmatrix}$ and then find a basis for each eigenspace.

Z-Z-Z END