23MCU519

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ABSTRACT ALGEBRA

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If a,b ∈ Z , a/b and b/a , then
(i) a = n b (ii) b = ma
(iii) a ≠ b (iv) a = ±b

2 If φ is a homomorphism of G into $\bar{G}$, then φ(x^{-1})

(i) $[\varphi(x)]^{-1}$ (ii) φ(x)
(iii) φ(e) (iv) 1

3 the order of disjoint cycles (1 2 3) (1 6 5 4 3)
(i) 2 (ii) 4
(iii) 3 (iv) 6

4 If R is a commutative ring with unit element, then R is a
(i) field (ii) Euclidean domain
(iii) integral domain (iv) none of these

5 If R is an integral domain then R[x] is ___
(i) integral domain (ii) field
(iii) ring (iv) Euclidean domain

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If a is a relatively prime to b but a / b c , then prove that a/c.

OR

b If a and b are integer, not both 0, then prove that (i) (a ,b) exists, (ii) (a, b) = $m_0 a + n_0 b$ for some $m_0, n_0 \in z$

7 a Prove that any subgroup of a cyclic group is itself a cyclic group.

OR

b If G is a finite group and N is a normal subgroup of G, then o(G/N) = o(G)/o(N).

8 a If G is a group , then A(G), the set of automorphism of G, is also a group.

OR

b Prove that any group of order p^2 is Abelian.

Cont...

9 a If φ is a homomorphism of R into R , then prove that (i) $\varphi(0) = 0$ (ii) $\varphi(-a) = -\varphi(a)$ for every $a \varepsilon R$.

OR

b If U is an ideal of the ring R then prove that R/U is a ring.

10 a If R be a Euclidean ring , Suppose that a ,b ,c ε R, a/ b c but (a, b) = 1 then prove that a / c .

OR

b If R is a commutative ring with unit element whose only ideals are {0} and R itself, then prove that R is a field.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Let $\sigma : S \rightarrow T$ and $\tau : T \rightarrow U$. Prove that $\sigma_0\tau$ is 1 -1 if each of σ *and* τ is 1-1.

OR

b Let G = $\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *and* $ad - bc \neq 0$ $a, b, c, d \varepsilon R\}$ Prove that G forms a group under matrix multiplication.

12 a Prove that HK is a subgroup of G if and only if HK = KH.

OR

b State and prove Cauchy's theorem for Abelian groups.

13 a State and prove Cayley's theorem.

OR

b Prove that every permutation is a product of is cycle.

14 a Prove that a finite integral domain is a field.

OR

b Prove that any homomorphism of a field is either an isomorphism or it takes each element into 'O'.

15 a If R is a commutative ring with unit element and M is a ideal, then prove that R/M is a field if and only if M is a maximal ideal.

OR

b Prove that every integral domain can be imbedded in a field.

Z-Z-Z END