TOTAL PAGES : 2
22MAU623

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Sixth Semester)

Branch – **MATHEMATICS**

GRAPH THEORY

Time: Three Hours   Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 A linear graph $G = (V, E)$ consists of a set of objects $V = \{v_1, v_2, ...\}$ called _____
(i) vertices (ii) edges
(iii) graphs (iv) lines

2 The degree of a tree is the ________ degree of a mode in the tree.
(i) Maximum (ii) Second minimum
(iii) Minimum (iv) Second maximum

3 A graph that cannot be drawn on a plane without a crossover between its edges is called ______
(i) coplanar (ii) nonplanar
(iii) complete (iv) isomorphic

4 The incidence matrix contains only two elements 0 and 1, such matrix is called a ______
(i) null matrix (ii) triangular matrix
(iii) binary matrix (iv) unit matrix

5 A diagraph that has non self-loop or parallel edges is called a ____
(i) simple digraphs (ii) Asymmetric diagraphs
(iii) symmetric diagraphs (iv) isographs

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Prove that the number of vertices of odd degree in a graph is always even.

OR

b Prove that, If a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices.

7 a Show that, there is one and only one path between every pair of vertices in a tree T.

OR

b If in a graph G there is one and only one path between every pair of vertices, then prove that G is a tree.

8 a Prove that, any simple planar graph can be embedded in a plane such that every edge is drawn as a straight line segment.

OR

b Show that, every tree with two or more vertices is 2-chromatic.

Cont...

9 a Prove, If B is a circuit matrix of a connected graph G with e edges and n vertices, rank of B = $e - n + 1$.

OR

b Prove that, the reduced incidence matrix of a tree is nonsingular.

10 a Prove that, The (i, j)th entry in X^r equals the number of different, directed edge sequences of r edges from the ith vertex to the jth.

OR

b Let A_f be the reduced incidence matrix of a connected digraph, then prove that the number of spanning trees in the graph equals the value of $\det(A_f, A_f^T)$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Explain – Briefly – Konisberg Bridge Problem.

OR

b Prove that a simple graph (i.e., a graph without parallel edges or self-loops) with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

12 a Prove that, every connected graph has atleast one spanning tree.

OR

b Prove that, A graph G with n vertices, $n - 1$ edges, and no circuits is connected.

13 a Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ regions.

OR

b Prove that the vertices of every planar graph can be properly colored with five colors.

14 a Prove that two graphs G_1 and G_2 are isomorophic if and only if their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and coloumns.

OR

b Let B and A be respectively the circuit matrix and the incidence matrix (of a self loop free graph) whose coloumns are arranged using the same order of edges. Then prove that every row of B is orthogonal to very row A, that is $A.B^T = B.A^T = 0 \ (mod\ 2)$, where superscript T denotes the transposed matrix.

15 a Prove that the determinant of every square submatrix of A, the incidence matrix of a digraph is 1, −1 or 0.

OR

b Prove that A simple digraph G of n vertices and $n - 1$ directed edges is an arborescence rooted at v_1 if and only if the (1,1) cofactor of $K(G)$ is equal to 1.

Z-Z-Z END