PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Second Semester)

Branch - **MATHEMATICS**

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Newton's Law of cooling is given by _____ a) $\frac{dT}{dt}=k(T-A)$ b) $\frac{dT}{dt}=-k(T-A)$ c) $\frac{dt}{dT}=-k(T-A)$ d) $\frac{dt}{dT}=k(T-A)$	K1	CO1
	2	A first-order differential equation of the form _____ is called Bernoulli equation. (a) $\frac{dy}{dx}+P(x)y=Q(x)y^n$ (b) $\frac{dy}{dx}-P(x)y=Q(x)y^n$ (c) $\frac{dy}{dx}+P(x)y^n=Q(x)y$ (d) $\frac{dy}{dx}-P(x)y^n=Q(x)y$	K1	CO1
2	3	The Wronskian of (cos x, sin x) is _____ a) cos x b) sin x c) 1 d) 0	K1	CO2
	4	The particular solution of $y''-4y=2e^{3x}$ is _____ a) $\frac{2}{3}e^{3x}$ b) $\frac{2}{5}e^{3x}$ c) $\frac{3}{5}e^{3x}$ d) e^{3x}	K2	CO2
3	5	A degenerate system may have either no solution or infinitely many _____ solutions. (a) dependent (b) unit (c) independent (d) zero	K2	CO3
	6	A linear first-order system is _____ if the functions are all identically zero. (a) non homogeneous (b) homogeneous (c) independent (d) dependent	K1	CO3
4	7	$L(t^{-1/2})$ = _____ a) $\sqrt{\frac{\pi}{s^2}}$ b) $\frac{\pi}{s}$ c) $\sqrt{\frac{\pi}{s^2}}$ d) $\sqrt{\frac{\pi}{s}}$	K2	CO4
	8	$L[u_a(t)]$ = _____ if a>0 (a) e^{as} (b) $\frac{e^{as}}{s}$ (c) e^{-as} (d) $\frac{e^{-as}}{s}$	K2	CO4
5	9	The basic circuit equation is _____ a) $L\frac{dI}{dt}+RI+\frac{1}{C}Q=E(t)$ b) $L\frac{dI}{dt}-RI+\frac{1}{C}Q=E(t)$ c) $L\frac{dI}{dt}-RI-\frac{1}{C}Q=E(t)$ d) $L\frac{dI}{dt}+RI+\frac{1}{C}Q+E(t)=0$	K1	CO5
	10	The convolution process associated with the Laplace Transform in time domain results into_____ a) Simple multiplication in complex frequency domain b) Simple division in complex frequency domain c) Simple multiplication in complex time domain d) Simple division in complex time domain	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Solve the initial value problem $\frac{dy}{dx} = y^2, y(1) = 2$	K2	CO1
		(OR)		
	11.b.	Solve the differential equation $\frac{dy}{dx} = (x+y+3)^2$		
2	12.a.	Evaluate the initial value problem $y''+2y' + y = 0$ given $y(0) = 5, y'(0) = -3$	K3	CO2
		(OR)		
	12.b.	Find the general solution of $9y^{(5)} - 6y^{(4)} + y^{(3)} = 0$		
3	13.a.	Solve the initial value problem $x' = -y, y' = (1.01)x - (0.2)y$ with given $x(0) = 0, y(0) = 1$	K2	CO3
		(OR)		
	13.b.	Derive a general solution of the system $(D^2 + 3)x - y = 0, -2x + (D^2 + 2)y = 0$		
4	14.a.	Evaluate L[cos (at) sinh (at)]	K3	CO4
		(OR)		
	14.b.	Evaluate the inverse Laplace transform of $\frac{s^2+1}{s^3-2s^2-8s}$		
5	15.a.	Obtain $L^{-1}\left[\frac{2s}{(s^2-1)^2}\right]$	K2	CO5
		(OR)		
	15.b.	Consider a mass on a spring with m = k = 1 and $x(0) = x'(0) = 0$. At each of the instants $t = 0, \pi, 2\pi, 3\pi, \ldots, n\pi, \ldots$ the mass is struck a hammer blow with a unit impulse. Determine the resulting motion.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	A 120-gallon (gal) tank initially contains 90 lb of salt dissolved in 90 gal of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full?	K4	CO1
2	17	Find the general solution of $y'' - 4y = 2e^{2x}$	K3	CO2
3	18	Solve the two-dimensional system $x' = -2y$ and $y' = \frac{1}{2}x$	K3	CO3
4	19	Using Laplace Transform, solve the initial value problem $x' + 4x = \sin 3t, x(0) = x'(0) = 0$	K4	CO4
5	20	Consider a mass–spring–dashpot system with m = 1, c = 4, and k = 20 in appropriate units. Suppose that the system is initially at rest at equilibrium ($x(0) = x'(0) = 0$) and that the mass is acted on beginning at time t = 0 by an external force f (t) - the square wave with amplitude 20 and period 2π. Find the position function f(t).	K4	CO5

Z-Z-Z END