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22MAU621

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Sixth Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If $f(z)=|z|^2$ then ____
(a) f is not differentiable (b) f is differentiable only at z=0
(c) f is differentiable at $z \neq 0$ (d) f is differentiable at every point

2 For the bilinear transformation $w = \frac{az+b}{cz+d}$, ______ is invariant.
(a) inverse point (b) refection point
(c) cross ratio (d) singular point

3 The value of $\frac{1}{2\pi i}\int_C \frac{z^2+5\ dz}{(z-3)}$ where C is the circle $|z|=4$
(a) 12 (b) 13 (c) 14 (d) 0

4 The Laurent's series expansion of $f(z)=z^2 e^{\frac{1}{z}}$ about z=0.
(a) $z^2+z+\frac{1}{2}+\frac{1}{3!z}+$.......... (b) $z^2-z+\frac{1}{2z}+\frac{1}{3!z}+$..........
(c) $z^2+z+\frac{1}{3}+\frac{1}{3!z}+$.......... (d) $z^2+z+\frac{1}{2z}+\frac{1}{3!z^2}+$..........

5 If $f(z)=\frac{e^z}{z^2}$ then Res$\{f(z);0\}$ is
(a) 0 (b) ∞ (c) z (d) 1

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If $f(z)$ *and* $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in the region.

OR

b Prove that the real and imaginary parts of an analytic function are harmonic functions.

7 a Show that the transformation $w=\frac{5-4z}{4z-2}$ maps the unit circle $|z|=1$ into a circle of radius unity and centre $\frac{-1}{2}$.

OR

b Prove that any bilinear transformation preserves cross ratio.

Cont...

8 a State and prove Cauchy's integral formula.

OR

b State and prove fundamental theorem of algebra.

9 a Expand cosz into Taylor's series about the point $z=\frac{\pi}{2}$.

OR

b Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in Laurent's series if $2<|z|<3$.

10 a State and prove Cauchy's Residue theorem.

OR

b Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

11 a Derive Cauchy's –Riemann equation in polar form.

OR

b Determine the analytic function $f(z)=u+iv$ if $u+v=\frac{\sin 2x}{\cosh 2y-\cos 2x}$

12 a (i) Find the image of the strip 2<x<3 under $w=\frac{1}{z}$

(ii) Find the bilinear transformation which maps the points $z_1=0, z_2=-i$, and $z_3=-1$ into $w_1=i, w_2=1$, and $w_3=0$ respectively.

OR

b Determine the bilinear transformation which maps 0,1, into $i,-1,-i$ respectively. Under this transformation show that the interior of the unit circle of the z-plane maps onto the half plane left to the v axis (left half of the w-plane).

13 a State and prove maximum modulus theorem.

OR

b State and prove Cauchy's theorem.

14 a Find the Laurent's series expansion of $\frac{-1}{(z-1)(z-2)}$ about

(i) $|z|=1$ (ii) $1<|z|<2$ (iii) $|z|>2$

OR

b State and prove Laurent's theorem.

15 a Evaluate $\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx \quad (a>b>0)$

OR

b State and prove Rouche's theorem.

Z-Z-Z END