PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch - MATHEMATICS

CALCULUS - I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry EQUAL marks   (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If $\vec{r}(t) = \langle t^3, ln(3-t), \sqrt{t} \rangle$ then the domain of $\vec{r}$ is ___. a) (0,3) b) [0.3] c) (0,3] d) [0,3)	K1	CO1
1	2	The length of the arc of the circular helix $\vec{r}(t) = cos\,t\vec{i} + sin\,t\vec{j} + t\vec{k}$ from the point (1,0,0) to the point $(1,0,2\pi)$ is _____? a) 2π b) $2\sqrt{2}\pi$ c) $\sqrt{2}\pi$ d) $1/\sqrt{2}\pi$	K2	CO1
2	3	If $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$ then $f(3,2) = $ ___. a) $\frac{\sqrt{6}}{2}$ b) $\frac{2}{\sqrt{6}}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{2}{\sqrt{3}}$	K1	CO2
2	4	The function $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ is discontinuous at the point ___ a) (0,0) b) (1,0) c) (0,1) d) (1,1)	K2	CO2
3	5	If $f(x,y) = xe^y$ then $\nabla f(2,0) = $ ___. a) $\langle 2,1 \rangle$ b) $\langle 1,2 \rangle$ c) $\langle 1,1 \rangle$ d) $\langle 2,2 \rangle$	K1	CO3
3	6	If (a,b) is a saddle point of the function $f(x,y)$ then ___ a) $D > 0$ b) $D < 0$ c) $D = 0$ d) none	K2	CO3
4	7	$\int_1^2 \int_0^3 x^2 y\,dxdy = $ ___. a) $\frac{27}{2}$ b) $-\frac{27}{2}$ c) $-\frac{27}{4}$ d) $\frac{27}{4}$	K1	CO4
4	8	If $R = [0,\pi/2] \times [0,\pi/2]$ then $\iint_R sin\,x\,cos\,y\,dA = $ ___ a) 0 b) 1 c) -1 d) 2	K2	CO4
5	9	The rectangular coordinates of the point with spherical coordinates $\left(2, \frac{2\pi}{3}, 1\right)$ is ___. a) $\left(1, \sqrt{3}, 1\right)$ b) $\left(1, \sqrt{3}, -1\right)$ c) $\left(-1, \sqrt{3}, -1\right)$ d) $\left(-1, \sqrt{3}, 1\right)$	K1	CO5
5	10	The equation of the surface $z^2 = x^2 + y^2$ in cylindrical coordinates is ___ a) $z = r^2$ b) $z^2 = r$ c) $z = r$ d) $z = \sqrt{r}$	K2	CO5

SECTION - B (35 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks   (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	For the curve $\vec{r}(t) = \sqrt{t}\vec{i} + (2-t)\vec{j}$, find $\vec{r}'(t)$ and locate the position of the vector $\vec{r}(1)$ and the tangent vector $\vec{r}'(1)$.	K2	CO1
1		(OR)		
1	11.b.	Show that the curvature of the circle of radius a is $1/a$.		
2	12.a.	Examine whether $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists if $f(x,y) = \frac{xy^2}{x^2+y^4}$.	K4	CO2
2		(OR)		
2	12.b.	The base radius and height of a right circular cone are measured as 10cm and 25cm, respectively, with a possible error in measurement of as much as cm in each. Use differentials to analyze the maximum error in the calculated volume of the cone.		

Module No.	Question No.	Question	K Level	CO
3	13.a.	If $z = f(x,y)$ has continuous second order partial derivatives and $x = r^2 + s^2$ and $y = 2rs$, construct $\frac{\partial z}{\partial r}$ and $\frac{\partial^2 z}{\partial r^2}$.	K3	CO3
		(OR)		
	13.b.	Suppose the temperature at a point (x,y,z) in space is given by $T(x,y,z) = \frac{80}{1+x^2+2y^2+3z^2}$, where T is measured in degree Celsius and x,y,z in meters. Identify the direction in which the temperature increase fastest at the point $(1,1,-2)$? Illustrate the maximum rate of increase?		
4	14.a.	Calculate the volume of the tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0, z = 0$.	K3	CO4
		(OR)		
	14.b.	Identify the mass and centre of mass of the triangular lamina with vertices $(0,0),(1,0),(2,0)$ if the density function is $\rho(x,y) = 1 + 3x + y$.		
5	15.a.	List the coordinates of the centre of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z; z = 0; x = 1$.	K4	CO5
		(OR)		
	15.b.	Deduce the value of the integral $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\,dz\,dy\,dx$.		

SECTION -C (30 Marks)
Answer ANY THREE questions
ALL questions carry EQUAL Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Deduce the equations of the normal plane and the osculating plane of the helix $\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}$ at the point $P\left(0,1,\frac{\pi}{2}\right)$,	K4	CO1
2	17	a) If $z = f(x,y) = x^2 + 3xy - y^2$ find the differential dz. b) If x changes from 2 to 2.05 and y changes from 3 to 2.96 compare the values of Δz and dz.	K4	CO2
3	18	Examine the maximum value of the function $f(x,y,z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.	K4	CO3
4	19	The manager of a movie theatre determines that the average time movie-goers wait in line to buy ticket for the week's film is 10 minutes and the average time they wait to buy popcorn is 5 minutes. Assuming that the waiting times are independent inspect the probability that a movie-goer waits a total of less than 20 minutes before taking his or her seat.	K4	CO4
5	20	Simplify $\iiint_E \sqrt{x^2 + z^2}\,dV$ where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.	K4	CO5

Z-Z-Z .   END