22MAU514

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours — Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$, then which of the following is true?
(i) $\inf A = \sup(-A)$ (ii) $\inf A = -\sup(-A)$
(iii) $\inf A = \sup(A)$ (iv) $\inf A = -\sup(A)$

2 Every infinite subset of a countable set A is _________.
(i) Countable (ii) Uncountable
(iii) Not denumerable (iv) Not enumerable

3 Which of the following set is connected?
(i) $[0,1) \cup (1,2]$ (ii) $[0,1] \cup [2,3]$
(iii) $[0,1] \cup [1,2]$ (iv) $(0,1) \cup (1,2)$

4 When a sequence $\{S_n\}$ of real number is said to be monotonically increasing?
(i) $S_n = S_{n+1}, (n = 1,2,3 \ldots)$ (ii) $S_n \leq S_{n+1}, (n = 1,2,3 \ldots)$
(iii) $S_n \geq S_{n+1}, (n = 1,2,3 \ldots)$ (iv) $S_n < S, (n = 1,2,3 \ldots)$

5 If f is continuous at every point of E, then f is said to be _____ on E.
(i) bounded (ii) unbounded
(iii) connected (iv) continuous

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If a and b are positive real numbers and n is a positive integer, then show that $(ab)^{1/n} = a^{1/n}b^{1/n}$.

OR

b Let z and w be complex numbers. Then show that
(i) $\overline{z+w} = \bar{z} + \bar{w}$,
(ii) $\overline{zw} = \bar{z}.\bar{w}$,
(iii) $z + \bar{z} = 2Re(z), z - \bar{z} = 2i\, Im(z)$,
(iv) $z\bar{z}$ is real and positive (except when $z = 0$).

7 a Let A be the set of all sequences whose elements are the digits 0 and 1. Then show that A is uncountable.

OR

b Prove that a set E is open if and only if its complement is closed.

8 a Prove that closed subsets of compact sets are compact.

OR

b State and prove Weierstrass theorem.

Cont...

22MAU514
Cont...

9 a State and Prove Ratio Test.

OR

b Show that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

10 a Let f be a continuous real function on the interval $[a, b]$. If $f(a) < f(b)$ and if c is a number such that $f(a) < c < f(b)$, then show that there exists a point $x \in (a, b)$ such that $f(x) = c$.

OR

b If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that $f(E)$ is connected.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Assume that S is an ordered set with the least-upper-bound property, $B \subset S$, B is not empty, and B is bounded below. Let L be the set of all lower bounds of B. Then prove that $\alpha = \sup L$ exists in S, and $\alpha = \inf B$.

OR

b If $a_1, a_2 \ldots\ldots, a_n$ and $b_1, b_2 \ldots\ldots, b_n$ are complex numbers, then show that $\left|\sum_{j=1}^{n} a_j \bar{b}_j\right|^2 \le \sum_{j=1}^{n} |a_j|^2 \sum_{j=1}^{n} |b_j|^2$.

12 a Let $\{E_n\}$, $n = 1,2,3 \ldots.$ be a sequence of countable sets and put $S = \bigcup_{n=1}^{\infty} E_n$. Then prove that S is countable.

OR

b If X is a metric space and $E \subset X$, then show that

(i) $\bar{E}$ is closed

(ii) $E = \bar{E}$ if and only if E is closed.

(iii) $\bar{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.

13 a A subset E of real line R^1 is connected if and only if it has the following property: If $x \in E$, $y \in E$ and $x < z < y$, then show that $z \in E$.

OR

b If a set E in R^k has one of the following three properties, then prove that it has the other two :

(i) E is closed and bounded

(ii) E is compact

(iii) Every infinite subset of E has a limit point in E.

14 a (i) If $\bar{E}$ is the closure of a set E in a metric space X, then prove that $diam\, \bar{E} = diam\, E$.

(ii) If K_n is a sequence of compact sets in X such that $K_n \supset K_{n+1}$ $(n = 1,2,3,..)$ and if $\lim_{n\to\infty} diam\, K_n = 0$, then prove that $\bigcap_{n=1}^{\infty} K_n$ consists of exactly one point.

OR

b Prove that $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$.

15 a Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

OR

b Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact.

Z-Z-Z END