PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Third Semester)

Branch – **MATHEMATICS**

PARTIAL DIFFERETIAL EQUATIONS & FOURIER TRANSFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

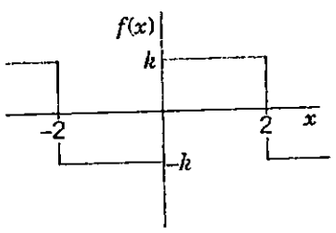
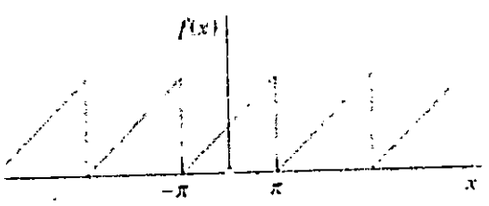
Module No.	Question No.	Question	K Level	CO
1	1	What is a non linear PDE of first order? a) $P(z)\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=0$ b) $P(z)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ c) $\left(\frac{\partial z}{\partial x}\right)^2-\left(\frac{\partial z}{\partial y}\right)^2=1$ d) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=1$	K1	CO1
	2	Form the partial differential equation by eliminating the constants from $z-ax+by+ab$. a) $z=x+y+xy$ b) $z=qx+py+xy$ c) $z=px+qy+pq$ d) $z=p+q+pq$	K2	CO1
2	3	Which is a Laplace equation in three dimensions? a) $u_t=k(u_{xx}+u_{yy}+u_{zz})$ b) $u_{xx}+u_{yy}+u_{zz}=0$ c) $u_{tt}=c^2(u_{xx}+u_{yy}+u_{zz})$ d) $u_t+uu_x=\mu u_{xx}$	K1	CO2
	4	Find the solution of the equation $(D^2+D'^2)u=0$. a) $\varphi_1(y+ix)+\varphi_2(y-ix)$ b) $\varphi_1(y-ix)-\varphi_2(y-ix)$ c) $\varphi_1(y+ix)-\varphi_2(y-ix)$ d) $\varphi_1(y+ix)+\varphi_2(y+ix)$	K2	CO2
3	5	If f is odd and $L=\pi$, then show that $f(x)$? a) $\sum_{n=1}^{\infty}a_n\cos nx$ b) $\sum_{n=1}^{\infty}b_n\sin nx$ c) $b_0+\sum_{n=1}^{\infty}b_n\sin nx$ d) $a_0+\sum_{n=1}^{\infty}a_n\cos nx$	K1	CO3
	6	In a digital age, which plays an important role? a) fast Fourier transform b) Fourier co-efficients c) Fourier series d) discrete Fourier transform.	K2	CO3
4	7	What is a Fourier sine integral? a) $f(x)=\int_0^1 B(w)\sin wx\,dw$ b) $f(x)=\int_{-\infty}^{\infty}B(w)\sin wx\,dw$ c) $f(x)=\int_0^{\infty}A(w)\sin wx\,dw$ d) $f(x)=\int_{-\infty}^{\infty}A(w)\sin wx\,dw$	K1	CO3
	8	Which is the Fourier sine transform of $f(x)$? a) $\hat{f}_s(w)=\int_0^{\infty}f(x)\sin wx\,dx$ b) $\hat{f}_s(w)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}f(x)\sin wx\,dx$ c) $\hat{f}_s(w)=\int_0^{\infty}B(w)\sin wx\,dx$ d) $\hat{f}_s(w)=\sqrt{\frac{2}{\pi}}\int_0^{1}f(x)\sin wx\,dx$	K2	CO3
5	9	What is the thermal conductivity k denotes, in the case for homogeneous material and non extreme temperatures. a) variable b) constants or variables c) constant d) temperature	K1	CO4
	10	If u_1 and u_2 are solutions of a what PDE in some region R, then $u=c_1u_1+c_2u_2$. a) homogeneous b) non homogeneous c) linear d) homogeneous linear	K2	CO4

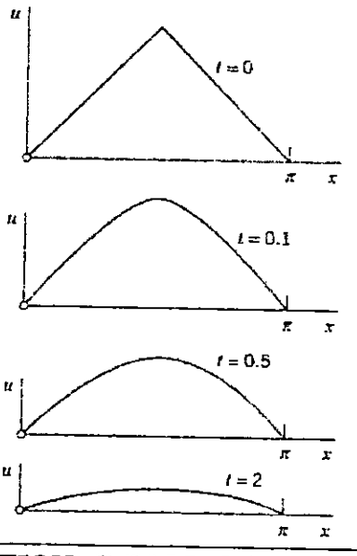
Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

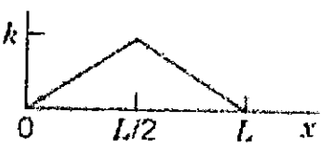
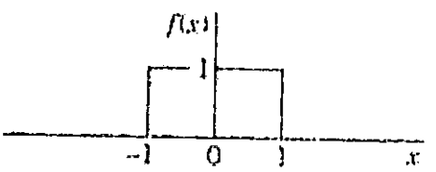
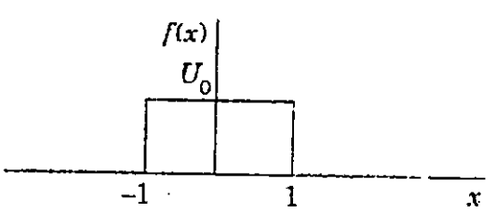
Module No.	Question No.	Question	K Level	CO
1	11.a.	Find Lagrange's method to solve the equation $\begin{vmatrix} x & y & z \\ \alpha & \beta & \gamma \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & -1 \end{vmatrix}$=0, where $z = z(x, y)$.	K2	CO1
		(OR)		
	11.b.	Find the complete integral of the PDE $p^2z^2 + q^2 = 1$.		
2	12.a.	Solve the Tricomi equation $u_{xx} + xu_{yy} = 0,\ x \neq 0$ for all x, y to canonical form.	K3	CO2
		(OR)		
	12.b.	Solve the following equation $(D - D' - 1)(D - D' - 2)u = e^{2x-y} + x$.		
3	13.a.	Apply the Fourier series to the function  $f(x) = \begin{cases} -k & if\ -2 < x < 0 \\ k & if\ 0 < x < 2 \end{cases}$ $p = 2L = 4,$ $L = 2$	K3	CO3
		(OR)		
	13.b.	Apply the Fourier series to the function  $f(x) = x + \pi \ \ if -\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.		
4	14.a.	Simplify $\mathfrak{F}_c(e^{-x})$.	K4	CO3
		(OR)		
	14.b.	State and prove Convolution theorem.		
5	15.a.	Explain the use of Fourier integrals.	K4	CO4
		(OR)		

	15.b.	Analyze the temperature in a laterally insulted bar of length L, whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x \ if\ 0 < x < \frac{L}{2} \\ L - x \ if \frac{L}{2} < x < L \end{cases}$ The given condition of the diagram is $L = \pi$. 		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Analyze the complete integral of the PDE: $z^2 = pq\ xy$.	K4	CO1
2	17	Solve the equation $u_{xx} + \frac{2N}{x}u_x = \frac{1}{a^2}u_{tt}$ where N and a are constants, is hyperbolic and obtain its canonical form.	K4	CO2
3	18	Simplify the two half-range expansions of the function  $f(x) = \begin{cases} \frac{2k}{L}\ x\ if\ 0 < x < \frac{L}{2} \\ \frac{2k}{L}\ (L - x)\ if \frac{L}{2} < x < L \end{cases}$	K4	CO3
4	19	Solve the Fourier integral representation to the function $f(x) = \begin{cases} 1\ if\ \lvert x\rvert < 1 \\ 0\ if\ \lvert x\rvert > 1 \end{cases}$ 	K4	CO3
5	20	Solve the heat problem in the Fourier transform the temperature in the infinite bar if the initial temperature is, $f(x) = \begin{cases} U_0 = const\ if\ \lvert x\rvert < 1 \\ 0 \qquad\quad if\ \lvert x\rvert > 1 \end{cases}$  By the method of convolution.	K4	CO4

Z-Z-Z END