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22MAU410N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fourth Semester)

Branch – **MATHEMATICS**

MODERN ALGEBRA

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Any two right cosets of H in G either are identical have or ____element in common. (a) One (b) two (c) no (d) three	K1	CO1
	2	The order of group S_3 is______ (a) 2 (b) 3 (c) 4 (d) 6	K2	CO1
2	3	The kernel of a homomorphism f: G → G′ is ______ (a) a sub group of G′ (b) a normal subgroup of G′ (c) a normal subgroup of G (d) {e}	K1	CO1
	4	In the quotient group $\frac{G}{N}$, N is _____ (a) any proper subgroup of G (b) a cyclic subgroup of G (c) a normal subgroup of G (d) a proper abelian subgroup of G	K2	CO2
3	5	The number of automorphisms of a cyclic group of order n is___ (a) $\varphi(n)$ (b) n (c) n^2 (d) 1	K1	CO2
	6	Consider an integer 23 such that 23 >= 3p for a 2p-cycle in a permutation group, then p is ___________ (a) odd prime (b) even prime (c) rational number (d) negative prime	K2	CO2
4	7	A ring R is called a Boolean ring if ∀a∈R (a) $a^2 = e$ (b) $a^2 = a$ (c) $a^2 = 0$ (d) $a^n = e$	K1	CO3
	8	Which of the following is not a ring ? (a) (Z, +, .) (b) (Q, +, .) (c) (R, +, .) (d) (R, ., +)	K2	CO3
5	9	The characteristic of (Q, +, ·) is (a) 0 (b) ∞ (c) 4 (d) 6	K1	CO4
	10	In the ring (Z_4, ⊕,⊗), ({0,2},⊕,⊗) is___ (a) not a subring (b) a subring with identity (c) subring without identity (d) a subfield	K2	CO4

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SECTION - B (35 Marks)

Answer ALL questions
ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	If G is a finite group whose order is a prime number p, then show that G is a cyclic group.	K2	CO1
		(OR)		
	11.b.	Show that there is a one-to-one correspondence between any two right cosets of H in G.		
2	12.a.	Analyze the statement HK is a subgroup of G if and only if $HK = KH$.	K4	CO1
		(OR)		
	12.b.	Analyze the statement N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.		
3	13.a.	Examine the statement every permutation is the product of its cycles.	K3	CO2
		(OR)		
	13.b.	Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then examine that $o(\phi(a)) = o(a)$.		
4	14.a.	If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.	K5	CO3
		(OR)		
	14.b.	Prove that a finite integral domain is a field.		
5	15.a.	Let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.	K5	CO4
		(OR)		
	15.b.	Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then Prove that R is a field.		

SECTION –C (30 Marks)

Answer ANY THREE questions
ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	If G is a group, then show that the following statements: i) The identity element of G is unique. ii) Every $a \in G$ has a unique inverse in G. iii) For every $a \in G$, $(a^{-1})^{-1} = a$. iv) For all $a, b \in G$, $(a.b)^{-1} = b^{-1}.a^{-1}$.	K2	CO1
2	17	State and prove Sylow's theorem for Abelian Groups.	K2	CO2
3	18	State and prove Cayley theorem.	K2	CO2
4	19	If ϕ is a homomorphism of R into R' with kernel $I(\phi)$, then prove that the following: i) $I(\phi)$is a subgroup of R under addition. ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.	K3	CO3
5	20	Analyze the statement Every integral domain can be imbedded in a field.	K4	CO4

Z-Z-Z END