22MAU515

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fifth Semester)

Branch – **MATHEMATICS**

LINEAR ALGEBRA

Time: Three Hours | Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. An example of a skew-symmetric matrix is _____
   (a) $\begin{pmatrix} 1 & -2 & -2 \\ 2 & 1 & -3 \\ 2 & 3 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$
   (c) $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

2. Let $f: C \to C$ be defined by $f(z) = \bar{z}$. Then ker f is ______
   (a) Φ (b) $\{0\}$ (c) $\{1\}$ (d) $\{i\}$

3. The standard inner product defined on $V_3(R)$ where $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ is _______
   (a) $\langle x, y\rangle = (x_1 + y_1)^2 + (x_2 + y_2)^2 + (x_3 + y_3)^2$
   (b) $\langle x, y\rangle = x_1y_1 + x_2y_2 + x_3y_3$
   (c) $\langle x, y\rangle = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$
   (d) $\langle x, y\rangle = (x_1^2 + y_1^2) + (x_2^2 + y_2^2) + (x_3^2 + y_3^2)$

4. The rank of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ is ______
   (a) 1 (b) 2 (c) 3 (d) 4

5. If the eigen values of a square matrix A are 1, 2, 3 then eigen values of A^2 are _____
   (a) 1, 4, 9 (b) 2, 4, 6 (c) $-1, -4, -9$ (d) $1, \frac{1}{2}, \frac{1}{3}$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a. Prove that the product of the two symmetric matrices is symmetric iff the matrices commute.

OR

b. Find $\begin{pmatrix} 1 & 2 & \sqrt{2} \\ \sqrt{3} & 4 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 1 & 2 \\ -\sqrt{3} & \sqrt[3]{5} & 7 \end{pmatrix}$.

7 a. If V is the internal direct sum of $U_1, U_2, \ldots\ldots U_n$ then Show that V is isomorphic to the external direct sum of $U_1, U_2, \ldots\ldots U_n$.

OR

b. If $v_1, v_2, \ldots\ldots v_n$ are in V then either they are linearly independent or some v_k, is a linear combination of the preceding ones, $v_1, v_2, \ldots\ldots v_{k-1}$.

8 a. Analyze the $dim_F V = m$ then $dim_F\, Hom\,(V, V) = m^2$.

OR

b. If V is a finite-dimensional inner product space and W is a subspace of V then Examine that $(W^T)^T = W$.

Cont...

9 a. Compute the rank of the matrix $\begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 3 & 1 & 1 & -1 & 2 \\ 4 & 0 & 1 & 0 & 3 \\ 9 & -1 & 2 & 3 & 7 \end{pmatrix}$

OR

b. Determine the characteristic roots of the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$

10 a. Apply the abstract concept of vector space if $T \epsilon A(V)$ and if $dim_F V = n$,then Prove that T can have at most n distinct characteristics in F.

OR

b. Compute the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a. Compute the inverse of $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{pmatrix}$.

OR

b. Prove that the matrix $U = \frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix}$ is unitary.

12 a. If V is finite-dimensional and if W is a subspace of V, then Prove that W is finite-dimensional, dim W $\leq$ dim V and dim v/W = dim V $-$ dim W.

OR

b. Prove that $F^{(n)}$ is isomorphic $F^{(m)}$ if and only if n=m.

13 a. If $u, v \in V$ then show that $|(u,v)| \leq \|u\|\|v\|$.

OR

b. Analyze the statement If V is finite-dimensional and $v \neq 0 \in V$, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

14 a. Verify the Cayley-Hamilton theorem for the matrix = $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

OR

b. Determine the characteristic vectors and the characteristic subspace corresponding to any one of the characteristic roots of the following matrix $\begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

15 a. Apply the abstract concept of vector space, if V is a finite dimensional over F, then $T \epsilon A(V)$ is regular iff T maps V onto V.

OR

b. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of q(T).

Z-Z-Z END