

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)
Branch - STATISTICS

REAL ANALYSIS AND LINEAR ALGEBRA

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is uniformly continuous in R ? (a) $f(x) = x^2$ (b) $f(x) = \sin(x)$ (c) $f(x) = e^x$ (d) $f(x) = \frac{1}{x}$ on $(0, \infty)$	K1	CO1
	2	The function $f(x) = \frac{x^2 - y^2}{x^2 + y^2}$ is (a) Continuous at (0,0) (b) Differentiable at (0,0) (c) Not continuous at (0,0) (d) Uniformly continuous on R^2	K2	CO2
2	3	The sequence $a_n = (-1)^n$ is (a) Convergent (b) Divergent (c) Oscillating but convergent (d) Monotonic	K1	CO1
	4	Which of the following tests is most useful in determining whether a series is conditionally convergent? (a) Root Test (b) Ratio Test (c) Alternating Series Test (d) Integral Test	K2	CO2
3	5	The Fundamental Theorem of Calculus relates the Riemann integral and (a) The derivative of a function (b) The sum of a series (c) The limits of sequences (d) The Taylor series expansion of a function	K1	CO1
	6	If $f(x) = x^2$ in the interval $[1, 3]$, what is the value of 'c' according to first mean value theorem. (a) $c = 2$ (b) $c = 2.5$ (c) $c = 3.2$ (d) $c = 4.3$	K2	CO2
4	7	The set of all vectors together with the two basic operations is (a) Orthogonal space (b) Vector space (c) Linear space (d) Metric space	K1	CO2
	8	The norm induced by the inner product $\langle u, v \rangle$ is defined as (a) $\ u\ = \langle u, v \rangle$ (b) $\ u\ = \langle u, u \rangle$ (c) $\ u\ = \sqrt{\langle u, v \rangle}$ (d) $\ u\ = \sqrt{\langle u, u \rangle}$	K2	CO1
5	9	The of a matrix A is the common value of the row rank of A and the column rank of A . (a) Nullity (b) Orthogonality (c) Rank (d) transformation	K1	CO1
	10	An $m \times n$ matrix A is said to be if its rank is m, (a) Row rank (b) Column rank (c) Full row rank (d) Full column rank	K2	CO2

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that if f is a continuous real valued function on the interval J , and if $f'(x) > 0$ for all x in J except possibly the end points of J , then f is strictly increasing on J .	K2	CO2
		(OR)		
	11.b.	Find the maxima and minima of the function $f(x,y) = x^2 + y^2 - 3x - 12y + 20$		
2	12.a.	If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L , then prove that $\{S_n^2\}_{n=1}^{\infty}$ converges to L^2	K2	CO4
		(OR)		
	12.b.	If b_n is a positive monotone decreasing function and if $\sum u_n$ is a convergent series, then prove that the series $\sum u_n b_n$ is also convergent.		
3	13.a.	If $f \in R[a,b]$ and $a < c < b$, then prove that $f \in R[a,c], *f \in R[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$	K1	CO2
		(OR)		
	13.b.	Prove that if a function defined on $[a,b]$ is [i] continuous on $[a,b]$ and [ii] derivable on $]a,b[$		
4	14.a.	Prove the intersection of any two subspaces S and T of a vector space V is a subspace of V .	K2	CO2
		(OR)		
	14.b.	Prove that every subspace S of a finite dimensional inner product space has an orthogonal basis. Moreover, any orthogonal subset of S can be extended to an orthogonal basis of S .		
5	15.a.	Let A and B be two matrices of order $m \times n$ and $n \times m$ respectively, where $m \leq n$. Then prove that $\chi_{BA}(\lambda) = \lambda^{n-m} \chi_{AB}(\lambda)$.	K1	CO1
		(OR)		
	15.b.	Prove that a number α is an Eigen value of A iff α is a characteristic root of A .		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Expand $x^2y + 3y - 2$ in powers of $x-1$ and $y+2$ using Taylor's series.	K2	CO2
2	17	Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+x^2}$, is uniformly convergent but not absolutely for all real values of x .	K2	CO1
3	18	If $f \in R[a,b]$ and λ is any real number, then prove that $\lambda f \in R[a,b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$	K1	CO2
4	19	Let $B = \{x_1, x_2, \dots, x_n\}$ be an orthogonal basis of an inner product space V . then prove that for any $x \in V$, we have $x = \sum_{j=1}^n \langle x, x_j \rangle x_j$	K2	CO2
5	20	Prove that any quadratic form $X^T A X$ can be reduced to a diagonal form by a non singular transformation of the variables.	K1	CO2

Z-Z-Z

END