

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - STATISTICS

STOCHASTIC PROCESSES

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is an example of a discrete-time stochastic process? a) Brownian motion b) Poisson process c) Markov chain d) Wiener process	K1	CO1
	2	The ergodic comes under _____ type random process? a) Stationary b) Non-stationary c) Both a and b d) None of the above	K2	CO1
2	3	Consider a Markov chain with state space $\{0, 1, 2\}$ and transition matrix $p = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$. Which of the following is False? a) The chain is reducible b) All states are positive recurrent c) The chain is aperiodic d) State 1 is accessible from other states	K1	CO1
	4	What is the stationary distribution $\pi = (\pi_1, \pi_2)$ for a two-state Markov chain with the following transition matrix? $P = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$ a) $\pi_1=0.5, \pi_2=0.5$ b) $\pi_1=0.25, \pi_2=0.75$ c) $\pi_1=0.42, \pi_2=0.58$ d) $\pi_1=0.4, \pi_2=0.6$	K2	CO1
3	5	In a Markov chain with continuous parameters, what does the transition probability $P(X(t)=j X(s)=i)$ depend on? a) The total time $t + s$ b) The time difference $t - s$ c) Only on the current state i d) It is always independent of time	K1	CO1
	6	What does the Kolmogorov forward equation describe? a) The probability of absorption over time b) The evolution of state probabilities over time c) The distribution of waiting times between transitions d) The relationship between stationary distributions	K2	CO1
4	7	Which of the following is NOT an assumption of the Poisson process? a) Events occur one at a time. b) The probability of more than one event in a small time interval is negligible. c) The time between consecutive events follows a uniform distribution. d) The number of events in non-overlapping intervals are independent.	K1	CO1

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4	8	In a birth-death process with birth rate $\lambda_n = n\lambda$ and death rate $\mu_n = n\mu$, what is the stationary distribution P_n ? a) $P_n = \lambda^n / \mu^n P_0$ b) $P_n = (\lambda/\mu)^n P_0$ c) $P_n = n\lambda / n\mu P_0$ d) $P_n = n (\lambda/\mu)^n P_0$	K2	CO1
5	9	In the Elementary Renewal Theorem, what is the asymptotic behaviour of the renewal function $m(t)$ as $t \rightarrow \infty$? a) $m(t) \sim t / E[T]$ b) $m(t) \sim E[T] \cdot t$ c) $m(t) \rightarrow \infty$ at a constant rate d) $m(t) \sim \sqrt{t} / E[T]$	K1	CO1
	10	Which renewal process models arrivals the arrival rate varies with time? a) Homogeneous Poisson process b) Non-homogeneous Poisson process c) Compound Poisson process d) Yule-Furry process	K2	CO1

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	What is a transition probability matrix (TPM)? State the properties of TPM. Detail the importance of TPM in the study of Markov Chains.	K2	CO2
		(OR)		
	11.b.	Let $\{y_n, n \geq 1\} \geq$ be a sequence of independent random variables with $P(Y_n = 1) = p = 1 - P(Y_n = -1)$. Let X_n be such that $X_0 = 0$ $X_{n+1} = X_n + Y_{n+1}$. <i>Examine if $\{X_n, n \geq 1\}$ is a Markov process.</i>		
2	12.a.	Define Markov Chain and n-step transition probability. Give one example of Markov Chain specifying its state space and parameter space.	K3	CO3
		(OR)		
	12.b.	Define the following terms: (i) Persistent state (ii) Recurrent State (iii) Period of a Markov Chain (iv) Mean first recurrence time		
3	13.a.	Explain whether random walk is a Markov Chain. Also define symmetric random walk.	K3	CO3
		(OR)		
	13.b.	Define random walk and write down the transition probability matrix for a random walk with absorbing barrier, reflections barrier and elastic barrier.		
4	14.a.	Define birth and death processes and obtain its differential-difference equations.	K4	CO4
		(OR)		
	14.b.	Obtain the expression $P_n(t)$ for birth process with usual notation.		
5	15.a.	Explain age and block replacement policies in renewal process.	K4	CO5
		(OR)		
	15.b.	Define renewal function. Show that the renewal function and the underlying renewal distribution can be determined from one another.		

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SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Define a Stochastic Process. Explain the classification of stochastic processes according to state space and time domain, giving an example for each type.	K3	CO3
2	17	Define an irreducible Markov Chain. Show that the states of an irreducible Markov Chain are all of the same type.	K3	CO4
3	18	Show that in a simple random walk with two reflecting barriers at 0 and K, the stationary distribution π_j is given by $\pi_j = \frac{1-p/q}{1-(p/q)^{K+1}} (p/q)^j \quad (j = 0, 1, 2, \dots, K)$ where $\pi_j = \lim_{n \rightarrow \infty} p_{ij}^{(n)}$ and $p_{j,j+1} = p$ and $p_{j,j-1} = 1 - p = q$	K4	CO3
4	19	Explain the concept of inter-arrival time in a Poisson process. How can you compute the probability density function of inter-arrival times? Provide formulae and an example to illustrate the calculation.	K4	CO4
5	20	Show that the renewal function satisfies the renewal equation $M(t) = F(t) + \int_0^t M(t-x) dF(x)$	K4	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - STATISTICS

MAJOR ELECTIVE COURSE – I DEMOGRAPHIC TECHNIQUES

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is NOT a technique used in demographic analysis? (a) Life table analysis (b) Crude birth rate calculation (c) Population projection (d) Climate change modeling	K1	CO1
	2	Infant Mortality Rate (IMR) measures the number of deaths of infants under the age of: (a) 6 months (b) 1 year (c) 5 years (d) 10 years	K2	CO2
2	3	In a life table, which symbol represents the probability of dying between ages x and $x+1$? (a) l_x (b) d_x (c) q_x (d) e_x	K1	CO1
	4	Makeham's Law of Mortality adds which component to Gompertz's law? (a) A constant force of mortality (b) A time-varying exponential term (c) A logarithmic function of age (d) A quadratic term for mortality	K2	CO2
3	5	What does the term "natality" refer to in demography? (a) The number of deaths in a population (b) The rate of migration in a population (c) The birth rate in a population (d) The mortality rate in a population	K1	CO1
	6	In Dandekar's modified Binomial model, the primary assumption about fertility is that: (a) Each birth is independent of previous births (b) Fertility is constant across all ages (c) The total number of births per woman is fixed (d) Fertility rates decrease exponentially with age	K2	CO1
4	7	The net migration rate is calculated as: (a) The difference between the number of immigrants and emigrants (b) The total number of immigrants divided by the total population (c) The difference between immigration and emigration per 1,000 population (d) The ratio of emigrants to immigrants	K1	CO2
	8	In the Lesile matrix, what do the elements in the first row of the matrix represent? (a) Age-specific survival rates (b) The total number of people in each age group (c) Age-specific mortality rates (d) Age-specific fertility rates	K2	CO1

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5	9	In a birth-death model, the expected population size at time t can be derived using: (a) Algebraic equations (b) A system of differential equations (c) Linear regression analysis (d) Random walk theory	K2	CO2
	10	Which of the following increases the extinction probability in a population? (a) Higher birth rates than death rates (b) Equal birth and death rates (c) Death rates greater than birth rates (d) Constant migration into the population	K1	CO1

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO																								
1	11.a.	Define demography. Also explain its nature and scope.	K1	CO2																								
	(OR)																											
	11.b.	Distinguish between Neo-natal and perinatal mortality rates.	K4	CO2																								
2	12.a.	Explain the maximum likelihood estimate of survival probability and its significance in demographic analysis.	K5	CO3																								
	(OR)																											
	12.b.	Discuss the Makeham's and Gompertz models of mortality and their applications in demographic studies.	K4	CO3																								
3	13.a.	Calculate G.F.R and T.F.R from the data given below: <table><tr><td><i>Child Bearing females</i></td><td>15-19</td><td>20-24</td><td>25-29</td><td>30-34</td><td>35-39</td><td>40-44</td><td>45-49</td></tr><tr><td><i>Number of Women ('000)</i></td><td>16.0</td><td>16.4</td><td>15.8</td><td>15.2</td><td>14.8</td><td>15.0</td><td>14.5</td></tr><tr><td><i>Total Births</i></td><td>260</td><td>2244</td><td>1894</td><td>1320</td><td>916</td><td>280</td><td>145</td></tr></table>	<i>Child Bearing females</i>	15-19	20-24	25-29	30-34	35-39	40-44	45-49	<i>Number of Women ('000)</i>	16.0	16.4	15.8	15.2	14.8	15.0	14.5	<i>Total Births</i>	260	2244	1894	1320	916	280	145	K3	CO3
		<i>Child Bearing females</i>	15-19	20-24	25-29	30-34	35-39	40-44	45-49																			
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		<i>Total Births</i>	260	2244	1894	1320	916	280	145																			
	Assume that the proportion of female births is 46:2.																											
	(OR)																											
13.b.	Explain the Poisson model of fertility and how Dandekar's modified Poisson model improves upon it.	K5	CO3																									
4	14.a.	State the meaning of migration and explain its key concepts in demographic studies.	K4	CO2																								
	(OR)																											
	14.b.	Elaborate Frejka's component method of population projection A small country has a population of 1,00,000 at the start of the year. The birth rate is 15 per 1,000, and the death rate is 8 per 1,000. Additionally, there is a net migration rate of 2 per 1,000. Using Frejka's component method of projection, estimate the population after one year.	K5	CO3																								
5	15.a.	Explain the system of differential equations used in modeling birth and death processes and its applications.	K5	CO3																								
	(OR)																											
	15.b.	Elaborate the concept of extinction probability in demographic models.	K5	CO3																								

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SECTION -C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO																				
1	16	Discuss the sources and quality of demographic data. How is the Chandrasekar-Deming Index used to assess data quality?	K4	CO3																				
2	17	Complete the life table of the population of a certain types of insects, x being the age in days and $l_x = 1000$ for $x = 0$: <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>q_x</td><td>0.120</td><td>0.005</td><td>0.010</td><td>0.050</td><td>0.100</td><td>0.500</td><td>0.800</td><td>0.900</td><td>0.950</td></tr></table>	x	0	1	2	3	4	5	6	7	8	q_x	0.120	0.005	0.010	0.050	0.100	0.500	0.800	0.900	0.950	K3	CO3
x	0	1	2	3	4	5	6	7	8															
q_x	0.120	0.005	0.010	0.050	0.100	0.500	0.800	0.900	0.950															
3	18	Define Natality and explain its concept and characteristics in demographic analysis.	K1	CO2																				
4	19	Explain the methods used to estimate migration rates. What are Hamilton's rates?	K5	CO3																				
5	20	Discuss the concept of birth and death processes as stochastic processes in demography.	K4	CO3																				

Z-Z-Z

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