

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - STATISTICS

STOCHASTIC PROCESSES

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is an example of a discrete-time stochastic process? a) Brownian motion b) Poisson process c) Markov chain d) Wiener process	K1	CO1
	2	The ergodic comes under _____ type random process? a) Stationary b) Non-stationary c) Both a and b d) None of the above	K2	CO1
2	3	Consider a Markov chain with state space $\{0, 1, 2\}$ and transition matrix $p = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$. Which of the following is False? a) The chain is reducible b) All states are positive recurrent c) The chain is aperiodic d) State 1 is accessible from other states	K1	CO1
	4	What is the stationary distribution $\pi = (\pi_1, \pi_2)$ for a two-state Markov chain with the following transition matrix? $P = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$ a) $\pi_1=0.5, \pi_2=0.5$ b) $\pi_1=0.25, \pi_2=0.75$ c) $\pi_1=0.42, \pi_2=0.58$ d) $\pi_1=0.4, \pi_2=0.6$	K2	CO1
3	5	In a Markov chain with continuous parameters, what does the transition probability $P(X(t)=j X(s)=i)$ depend on? a) The total time $t + s$ b) The time difference $t - s$ c) Only on the current state i d) It is always independent of time	K1	CO1
	6	What does the Kolmogorov forward equation describe? a) The probability of absorption over time b) The evolution of state probabilities over time c) The distribution of waiting times between transitions d) The relationship between stationary distributions	K2	CO1
4	7	Which of the following is NOT an assumption of the Poisson process? a) Events occur one at a time. b) The probability of more than one event in a small time interval is negligible. c) The time between consecutive events follows a uniform distribution. d) The number of events in non-overlapping intervals are independent.	K1	CO1

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4	8	In a birth-death process with birth rate $\lambda_n = n\lambda$ and death rate $\mu_n = n\mu$, what is the stationary distribution P_n ? a) $P_n = \lambda^n / \mu^n P_0$ b) $P_n = (\lambda/\mu)^n P_0$ c) $P_n = n\lambda / n\mu P_0$ d) $P_n = n (\lambda/\mu)^n P_0$	K2	CO1
5	9	In the Elementary Renewal Theorem, what is the asymptotic behaviour of the renewal function $m(t)$ as $t \rightarrow \infty$? a) $m(t) \sim t / E[T]$ b) $m(t) \sim E[T] \cdot t$ c) $m(t) \rightarrow \infty$ at a constant rate d) $m(t) \sim \sqrt{t} / E[T]$	K1	CO1
	10	Which renewal process models arrivals the arrival rate varies with time? a) Homogeneous Poisson process b) Non-homogeneous Poisson process c) Compound Poisson process d) Yule-Furry process	K2	CO1

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	What is a transition probability matrix (TPM)? State the properties of TPM. Detail the importance of TPM in the study of Markov Chains.	K2	CO2
		(OR)		
	11.b.	Let $\{y_n, n \geq 1\} \geq$ be a sequence of independent random variables with $P(Y_n = 1) = p = 1 - P(Y_n = -1)$. Let X_n be such that $X_0 = 0$ $X_{n+1} = X_n + Y_{n+1}$. <i>Examine if $\{X_n, n \geq 1\}$ is a Markov process.</i>		
2	12.a.	Define Markov Chain and n-step transition probability. Give one example of Markov Chain specifying its state space and parameter space.	K3	CO3
		(OR)		
	12.b.	Define the following terms: (i) Persistent state (ii) Recurrent State (iii) Period of a Markov Chain (iv) Mean first recurrence time		
3	13.a.	Explain whether random walk is a Markov Chain. Also define symmetric random walk.	K3	CO3
		(OR)		
	13.b.	Define random walk and write down the transition probability matrix for a random walk with absorbing barrier, reflections barrier and elastic barrier.		
4	14.a.	Define birth and death processes and obtain its differential-difference equations.	K4	CO4
		(OR)		
	14.b.	Obtain the expression $P_n(t)$ for birth process with usual notation.		
5	15.a.	Explain age and block replacement policies in renewal process.	K4	CO5
		(OR)		
	15.b.	Define renewal function. Show that the renewal function and the underlying renewal distribution can be determined from one another.		

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SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Define a Stochastic Process. Explain the classification of stochastic processes according to state space and time domain, giving an example for each type.	K3	CO3
2	17	Define an irreducible Markov Chain. Show that the states of an irreducible Markov Chain are all of the same type.	K3	CO4
3	18	Show that in a simple random walk with two reflecting barriers at 0 and K, the stationary distribution π_j is given by $\pi_j = \frac{1-p/q}{1-(p/q)^{K+1}} (p/q)^j \quad (j = 0, 1, 2, \dots, K)$ where $\pi_j = \lim_{n \rightarrow \infty} p_{ij}^{(n)}$ and $p_{j,j+1} = p$ and $p_{j,j-1} = 1 - p = q$	K4	CO3
4	19	Explain the concept of inter-arrival time in a Poisson process. How can you compute the probability density function of inter-arrival times? Provide formulae and an example to illustrate the calculation.	K4	CO4
5	20	Show that the renewal function satisfies the renewal equation $M(t) = F(t) + \int_0^t M(t-x) dF(x)$	K4	CO5

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END