

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – STATISTICS

ADVANCED PROBABILITY THEORY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Conditional variance of a variable X for given Y=y in terms of conditional expectation can be expressed as: (a) $V(X/Y = y) = E(Y^2/X = x) - [E(X/Y = y)]^2$ (b) $V(X/Y = y) = [E(Y/X = x)]^2 - [E(X^2/Y = y)]$ (c) $V(X/Y = y) = E(X^2/X = x) - [E(X/Y = y)]^2$ (d) $V(X/Y = y) = E(X^2/Y = y) - [E(X/Y = y)]^2$	K1	CO1
	2	$P(X - m_i \geq k\sigma) \leq \frac{1}{k^2}$, for every positive k, denotes _____ inequality. (a) Chebyshev (b) Holders (c) Jensen (d) Markov	K2	CO1
2	3	The characteristic function of a Cauchy distribution $\phi(u), u \in R$ is _____. (a) u (b) 1/ u (c) exp(u) (d) exp(- u)	K1	CO2
	4	Distribution function F of a random variable and its _____ function determine each other. (a) density (b) characteristic (c) probability (d) continuity	K2	CO2
3	5	If A and B^c are independent then $P(AB^c) =$ _____. (a) $PA + PB^c$ (b) $PA - PB^c$ (c) $PA \cdot PB^c$ (d) PA/PB^c	K1	CO3
	6	Tail events of a sequence of independent random variables have for their probability either zero or one where tail functions are almost surely degenerate is the statement of _____. (a) Kolmogorov 0-1 law (b) Borel 0-1 law (c) Borel –Cantelli lemma (d) weak law of large numbers.	K2	CO3
4	7	Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. then $aX_n \xrightarrow{P} ?$ (where a is real) (a) a (b) X_n (c) Y_n (d) aX	K1	CO4
	8	A sequence of random variables converges almost surely to a random variable if the sequence converges _____. (a) mutually almost surely (b) pairwise almost surely (c) mutually (d) pairwise	K2	CO4
5	9	In proving the De Moivre-Laplace theorem, the sequence $\{Y_n\}$ of standardized random variables is from _____ distribution. (a) Poisson (b) Laplace (c) Binomial (d) Normal	K1	CO5
	10	If the sum of random variables has an asymptotically normal distribution, then we say that it satisfies _____ theorem. (a) central limit (b) Cauchy (c) Levy (d) Khinchine	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Obtain mean of Hypergeometric distribution.	K4	CO1
		(OR)		
	11.b.	Construct Jensen's inequality.	K5	
2	12.a.	If X follows standard normal variate, then find the characteristic function of X.	K3	CO2
		(OR)		
	12.b.	Build the necessary condition of Bochner's theorem.	K2	
3	13.a.	Let X and Y be two independent random variables taking the values +1 and -1 with probability $\frac{1}{2}$ each. If $Z=XY$, prove that X, Y, Z are pairwise independent, but not mutually independent.	K3	CO3
		(OR)		
	13.b.	Prove that sub-classes of independent classes are independent.	K5	
4	14.a.	State and prove Helly Bray's lemma.	K5	CO4
		(OR)		
	14.b.	Discuss the convergence of random variables and probability.	K6	
5	15.a.	Explain Kolmogorov's strong law of large numbers.	K2	CO5
		(OR)		
	15.b.	Discuss the statements of (a) Liapounov's and (b) Lindeberg Feller's central theorem with applications.	K6	

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove C_r inequality.	K5	CO1
2	17	Derive the inversion formula based on characteristic functions.	K3	CO2
3	18	State and prove Borel-Cantelli Lemma.	K5	CO3
4	19	Defend: $X_n \xrightarrow{r} X \Rightarrow E X_n ^r \rightarrow E X ^r$.	K4	CO4
5	20	State and prove Lindeberg-Levy's Theorem.	K5	CO5

Z-Z-Z

END