

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc(SS) DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – SOFTWARE SYSTEMS(Five Years Integrated)

CALCULUS AND ITS APPLICATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 x 1 = 10)

- 1 Find $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
(i) 0 (ii) 2 (iii) 4 (iv) 3
- 2 What is the value of definite integral $\int_0^2 (2x + 1) dx$
(i) 4 (ii) 5 (iii) 6 (iv) 7
- 3 Which of the following is the correct value of series : $\sum_{n=0}^{\infty} \frac{4}{2^n}$
(i) 2 (ii) 4 (iii) 8 (iv) diverges
- 4 Identify the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ does this series converge?
(i) Yes, by the ratio test (ii) No, by the ratio test
(iii) Yes, by the root test (iv) No, by the root test
- 5 What is $\frac{dy}{dx}$ if $y = \cos(4x^2 + 3)$ using chain rule
(i) $-8x \sin(4x^2 + 3)$ (ii) $8x \sin(4x^2 + 3)$
(iii) $4x \sin(4x^2 + 3)$ (iv) $-4x \sin(4x^2 + 3)$
- 6 When the function $f(x, y) = x^3 + y^3 - 12x - 3y + 20$. Determine the nature of the critical point at (6, -1)
(i) local maximum (ii) local minimum
(iii) saddle point (iv) cannot be determined
- 7 Find the general solution of $xy' = 2y + x^3 e^x$
(i) $y = Ce^{2x} + \frac{x^3 e^x}{x^2}$ (ii) $y = Ce^{2x} + x^2 e^x - x^2$
(iii) $y = Ce^{2x} + x^3 e^x$ (iv) $y = Cx^2 + e^x$
- 8 Identify the given ODE $(D^5 + 2D^3 + D)y = 0$
(i) $y(x) = C_1 + C_2 e^x + C_3 e^x$ (ii) $y(x) = C_1 + C_2 \cos(x) + C_3 \sin(x)$
(iii) $y(x) = C_1 + C_2 x + C_3 x^2$ (iv) $y(x) = C_1 e^{ix} + C_2 e^{ix} + C_3$
- 9 Choose whether function $f(x) = x \cot(x)$ is an even function, odd function or neither
(i) Even function (ii) Odd function
(iii) Neither even nor odd (iv) Both even and odd
- 10 What is the fundamental period of the function $f(x) = \cos(x)$
(i) π (ii) 2π (iii) 4π (iv) $\frac{2\pi}{3}$

SECTION - B (25 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 5 = 25)

- 11 a Evaluate if $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following
(i) $f(g(0))$ (ii) $f(f(x))$ (iii) $g(f(x))$
OR
- b Solve $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$
- 12 a State and prove the monotonic sequence theorem.
OR

Cont...

Q.no 12 Cont....

- b Define ratio test. Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{2^{n+5}}{3^n}$

- 13 a Analyze $\lim_{(x,y) \rightarrow (0,0)} \frac{4xy^2}{x^2+y^2}$ if it exists.

OR

- b Determine the linearization of $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point $(x_0, y_0, z_0) = (2, 1, 0)$. Find an upper bound for the error incurred in replacing f by L on the rectangular region $R: |x - 2| \leq 0.01, |y - 1| \leq 0.02, |z| \leq 0.01$.

- 14 a Evaluate an integrating factor and solve the initial value problem $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, y(0) = -1$

OR

- b Solve: $2xyy' = y^2 - x^2$

- 15 a Illustrate the Fourier series of the function

$$f(x) = \begin{cases} -k, & \text{if } -2 < x < 0 \\ k, & \text{if } 0 < x < 2 \end{cases} \quad p = 2L = 4, L = 2.$$

OR

- b Discuss: i) periodic function ii) Even function of period 2π

SECTION -C (40 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 8 = 40)

- 16 a Evaluate that $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$, prove that

$$\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$$

OR

- b Determine the limiting value of lower sum approximations to the area of the region R of $y = 1 - x^2$ and above the interval $[0, 1]$ on the x -axis using equal-width rectangles whose widths approaches zero and whose number approaches infinity.

- 17 a Justify the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converge?
If so, find $\lim_{n \rightarrow \infty} a_n$

OR

- b Enumerate that the Taylor series generated by $f(x) = e^x$ at $x = 0$ converges to $f(x)$ for every real value of x .

- 18 a (i) Solve the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $v = 2i - 3j + 6k$.

(ii) In what directions does f change most rapidly at P_0 and what are the rates of change in these directions?

OR

- b Evaluate the point $p(x, y, z)$ on the plane $2x + y - z - 5 = 0$ that is closest to the origin.

- 19 a Analyze that y is a solution of the ODE. Determine from y particular solution of the IVP $y' + 4y = 1.4, y = ce^{-4x} + 0.35, y(0) = 2$

OR

- b Solve the IVP on any open interval I on the positive x -axis containing $x = 1$ $x^3y''' - 3x^2y'' + 6xy' - 6y = 0, y(1) = 2, y'(1) = 1, y''(1) = -4$

- 20 a Develop the two half-range expansions of the function

$$f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x), & \text{if } \frac{L}{2} < x < L \end{cases}$$

OR

- b Enumerate that $f = \text{const}$ is periodic with any period but has no fundamental period.

Z-Z-Z

END