

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – SOFTWARE SYSTEMS (Five years Integrated)

APPLIED LINEAR ALGEBRA

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

1. If two linear equations are given as $2x+3y=5$ and $4x+6y=10$, what is the nature of their solution?
(i) No solution (ii) Unique solution
(iii) Infinitely many solutions (iv) None of the above
2. If a set of vectors is linearly independent, this means:
(i) No vector in the set can be written as a linear combination of the others
(ii) Every vector in the set can be written as a linear combination of the others
(iii) The vectors span the entire vector space
(iv) The set contains the zero vector
3. In an inner product space, two vectors u and v are orthonormal if:
(i) They are orthogonal, and their norms are 1.
(ii) They are parallel, and their norms are 1.
(iv) Their inner product is greater than 1.
(iv) Their norms are both equal to 0.
4. What is the kernel of a linear transformation?
(i) The set of vectors that the transformation maps to the zero vector
(ii) The set of vectors that the transformation maps to the identity vector.
(iii) The set of vectors that are scaled by the transformation
(iv) The set of all possible images of the transformation.
5. Which of the following statements is true about eigenvalues of a matrix?
(i) Eigenvalues are always positive
(ii) Eigenvalues can be real or complex numbers
(iii) Eigenvalues must always be distinct
(iv) Eigenvalues are always integers.

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 3 = 15)

- 6 a Given $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find $4u$, $(-3)v$ and $4u + (-3)v$.
OR
b When we say the rectangular matrix is in echelon form?
- 7 a Define vector space and give one example.
OR
b Use the Wronskian to show that $f_1 = 1$, $f_2 = e^x$ and $f_3 = e^{2x}$ are linearly independent.
- 8 a State and prove Cauchy-Schwarz inequality.
OR
b State and prove projection theorem.
- 9 a If $T: V \rightarrow W$ is a linear transformation, then prove the following:
i) $T(0) = 0$
ii) $T(u - v) = T(u) - T(v)$ for all u and v in V .
OR

Cont...

Q.no.9 Cont...

- 9 b Let $T: P_1 \rightarrow P_2$ be the linear transformation defined by $T(p(x)) = xp(x)$ Find the matrix for T with respect to the standard bases $B = \{u_1, u_2\}$, and $B' = \{v_1, v_2, v_3\}$ where $u_1 = 1, u_2 = x; v_1 = 1, v_2 = x, v_3 = x^2$.
- 10 a Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are u and v eigen vectors?
OR
- b If $v_1, v_2, \dots, v_r$ are eigen vectors that correspond to distinct eigen values $\mu_1, \mu_2, \dots, \mu_r$ of an $n \times n$ matrix A , then prove that the set $\{v_1, v_2, \dots, v_r\}$ is linearly independent.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

- 11 a Determine if the following homogeneous system has a nontrivial solution, then find the solution set.
 $3x_1 + 5x_2 - 4x_3 = 0; -3x_1 - 2x_2 + 4x_3 = 0; 6x_1 + x_2 - 8x_3 = 0;$
OR
- b Solve the following system of equations if system is consistent.
 $x_2 - 4x_3 = 8; 2x_1 - 3x_2 + 2x_3 = 1; 5x_1 - 8x_2 + 7x_3 = 1;$
- 12 a Determine whether $v_1 = (1,1,2), v_2 = (1,0,1)$ and $v_3 = (2,1,3)$ span the vector space R^3

OR

- b Find the rank and nullity of the matrix $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$

- 13 a i) Show that the vectors $w_1 = (0,2,0), w_2 = (3,0,3), w_3 = (-4,0,4)$ form an orthogonal basis for R^3 with the Euclidean inner product, and use that basis to find an orthonormal basis by normalizing each vector.
ii) Express the vector $u = (1,2,4)$ as a linear combination of the orthonormal basis vectors obtained in part (i).

OR

- b Find the QR-Decomposition of 3×3 matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

- 14 a Prove that every n -dimensional vector space is isomorphic to R^n .

OR

- b Let $T: P_2 \rightarrow P_2$ be the linear operator defined by $T(p(x)) = p(3x - 5)$ that is, $T(c_0 + c_1x + c_2x^2) = c_0 + c_1(3x - 5) + c_2(3x - 5)^2$
i) Find $[T]_B$ relative to the basis $B = \{1, x, x^2\}$.
ii) Use the indirect procedure to compute $T(1 + 2x + 3x^2)$
iii) Check the result in (ii) by computing $T(1 + 2x + 3x^2)$ directly.

- 15 a Define characteristic equation and Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

OR

- b Diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}$