

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch - MATHEMATICS

MEASURE THEORY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$m^*(A) \geq$ i) 0 ii) ∞ iii) 1 iv) A	K1	CO1
	2	If $F \in \mathcal{M}$ and ____ then G is measurable. i) $m^*(F \Delta G) = F$ ii) $m^*(F \Delta G) = G$ iii) $m^*(F \Delta G) = 1$ iv) $m^*(F \Delta G) = 0$	K2	CO1
2	3	If f is a non-negative measurable function, then $f = 0$ a. e if and only if $\int f dx =$ i) x ii) 1 iii) f iv) 0	K1	CO2
	4	If f is a measurable function such that at least one of $\int f^+ dx$, $\int f^- dx$ is finite then, $\int f dx =$ i) $\int f^+ dx - \int f^- dx$ ii) $\int f^+ dx - \int f^- dx$ iii) $\int f^+ dx + \int f^- dx$ iv) $\int f^- dx - \int f^+ dx$	K2	CO2
3	5	A class of sets $\mathcal{R}$ of some fixed space is called a ____ if whenever $E \in \mathcal{R}$ then $E \cup F$ and $E - F$ belongs to $\mathcal{R}$ i) integral domain ii) Group iii) Ring iv) Field	K1	CO3
	6	A pair $[[X, \mathcal{S}]]$ where $\mathcal{S}$ is a σ -algebra of subsets of a space X , is called a ____. i) measurable space ii) measure space iii) metric space iv) Euclidean space	K2	CO3
4	7	$L^\infty(X, \mu)$ is a ____ over the real numbers i) metric space ii) vector space iii) Euclidean space iv) measurable space	K1	CO4
	8	$\ f + g\ _\infty \leq$ i) $\ f\ _\infty$ ii) $\ g\ _\infty$ iii) $\ f\ _\infty + \ g\ _\infty$ iv) $\ f\ _\infty - \ g\ _\infty$	K2	CO4
5	9	If $\phi(E) = \int_E f d\mu$ where $\int f d\mu$ is defined then ϕ is a ____ i) signed measure ii) unsigned measure iii) relative measure iv) no measure	K1	CO5
	10	If μ is a measure, $\int f d\mu$ exists and $v(E) = \int_E f d\mu$ then ____ i) $v \gg \mu$ ii) $v \ll \mu$ iii) $v^+ \ll \mu$ iv) $v^+ \gg \mu$	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	For any sequence of sets $\{E_i\}$, examine $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$	K4	CO1
		(OR)		
	11.b.	Let f be a measurable function and B a Borel set; then inspect $f^{-1}(B)$ is a measurable set.		
2	12.a.	Explain: Let f be a non-negative measurable function. Then there exists a sequence $\{\varphi_n\}$ of measurable simple functions such that for each x , $\varphi_n(x) \uparrow f(x)$	K2	CO2
		(OR)		
	12.b.	State and Prove Lebesgue's Dominated Convergence Theorem		
3	13.a.	Examine: Let $\{A_i\}$ be a sequence in a ring $\mathcal{R}$ then there is a sequence $\{B_i\}$ of disjoint sets of $\mathcal{R}$ such that $B_i \subseteq A_i$ for each i and $\bigcup_{i=1}^N A_i = \bigcup_{i=1}^N B_i$ for each N , so that $\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} B_i$	K4	CO3
		(OR)		
	13.b.	Inspect: If μ is a measure on a σ -ring $\mathcal{S}$, then the class $\bar{\mathcal{S}}$ of sets of the form $E \Delta N$ for any sets E, N such that $E \in \mathcal{S}$ while N is contained in some set in $\mathcal{S}$ of measure zero is a σ -ring, and the set function $\bar{\mu}$ defined by $\bar{\mu}(E \Delta N) = \mu(E)$ is complete measure on $\bar{\mathcal{S}}$.		
4	14.a.	Explain: Let $f, g \in L^p(\mu)$ and let a, b be constants; then $af + bg \in L^p(\mu)$	K2	CO4
		(OR)		
	14.b.	State and Prove Jensen's Inequality		
5	15.a.	Build: A countable union of sets positive with respect to a signed measure ν is a positive set.	K3	CO5
		(OR)		
	15.b.	Build: Let ν be a signed measure on $[X, \mathcal{S}]$. Then there exist measures ν^+ and ν^- on $[X, \mathcal{S}]$ such that $\nu = \nu^+ - \nu^-$ and $\nu^+ \perp \nu^-$. The measures ν^+ and ν^- are uniquely defined by ν and $\nu = \nu^+ - \nu^-$ is said to be the Jordan decomposition of ν		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Examine: The outer measure of an interval equals its length.	K4	CO1
2	17	State and Prove Fatou's Lemma	K5	CO2
3	18	Explain: Let μ^* be an outer measure on $\mathcal{K}(\mathcal{R})$ and let $\mathcal{S}^*$ denote the class of μ^* measurable sets. Then $\mathcal{S}^*$ is a σ ring and μ^* restricted to $\mathcal{S}^*$ is complete measure.	K5	CO3
4	19	Explain: A differentiable function ψ is convex on (a, b) if and only if ψ' is a monotone increasing function. If ψ'' exists on (a, b) then ψ is convex if, and only if $\psi'' \geq 0$ on (a, b) and strictly convex if $\psi'' > 0$ on (a, b)	K5	CO4
5	20	Explain: Let ν be a signed measure on $[X, \mathcal{S}]$. Let $E \in \mathcal{S}$ and $\nu(E) > 0$. Then there exists A , a set positive with respect to ν , such that $A \subseteq E$ and $\nu(A) > 0$	K5	CO5

Z-Z-Z

END