

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – MATHEMATICS

MATHEMATICAL STATISTICS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module	Question No.	Question	K Level	CO								
1	1	The table of probability distribution of a random variable X is given below. <table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(x_i)$</td><td>$\frac{1}{3}$</td><td>K</td><td>$\frac{1}{6}$</td></tr></table> Then the expectation of X is _____ (a) $\frac{5}{6}$ (b) 1 (c) 0 (d) $1/6$	x_i	0	1	2	$P(x_i)$	$\frac{1}{3}$	K	$\frac{1}{6}$	K1	CO1
	x_i	0	1	2								
$P(x_i)$	$\frac{1}{3}$	K	$\frac{1}{6}$									
2	2	The value of $\phi(0) =$ _____. (a) 1 (b) -1 (c) 0 (d) ∞	K2	CO1								
2	3	In a binomial distribution the mean is 4 and the variance is $\frac{8}{3}$. Then the values of n and p are respectively _____ (a) $12, \frac{1}{3}$ (b) $12, \frac{2}{3}$ (c) $16, \frac{1}{3}$ (d) $12, \frac{1}{9}$	K1	CO2								
	4	In a poisson distribution , if $P(X=0)=k$ then the variance is _____ (a) e^k (b) e^{-k} (c) $\log_e k$ (d) $\log_e \left(\frac{1}{k}\right)$	K2	CO2								
3	5	The sequence of random variables $\{Z_n\} = \{X_n - X\}$ is stochastically convergent to _____. (a) 1 (b) $-\infty$ (c) ∞ (d) 0	K1	CO3								
	6	The random variable X_n has an asymptotically normal distribution with standard deviation is _____. (a) npq (b) np (c) $\sqrt{npq}$ (d) $\sqrt{np}$	K2	CO3								
4	7	If the process is real and $m = 0, \sigma^2 = 1$, then $R(\tau)$ is called _____ function. (a) Correlation (b) Regression (c) Moment (d) Stationary	K1	CO4								
	8	If j is transient, then as $n \rightarrow \infty, p_{jj}^{(n)} \rightarrow$ _____. (a) 1 (b) 0 (c) ∞ (d) -1	K2	CO4								
5	9	Find the number of degrees of freedom in a 3×3 contingency table is : (a) 8 (b) 4 (c) 3 (d) 5	K1	CO5								
	10	Analysis of variance (ANOVA) performs which type of test? (a) F-test (b) t-test (c) z-test (d) xy-test	K2	CO5								

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11a.	State and prove Chebyshev inequality.	K5	CO1
		(OR)		
	11b.	The random variable X can take on the values $x_1 = -1$ and $x_2 = 1$ with probabilities $P(X = -1) = P(X = 1) = 0.5$. Determine the characteristic function.		

Cont...

Cont...

2	12.a.	The random variable X has the distribution $N(1; 2)$. Find the probability that X is greater than 3.	K1	CO2
	(OR)			
	12.b.	Obtain the moments of the normal distribution.		
3	13.a.	We throw a coin $n = 100$ times. We assign the number 1 to the appearance of heads & the number 0 to the appearance of tails. The probability of each of these events is equal to $p = q = 0.5$. what is the probability that heads will appear more than 50 times & less than 60 times ?	K1	CO3
	(OR)			
	13.b.	Let $F_n(x)$ ($n = 1, 2, \dots$) be the distribution function of the random variable X_n . Prove that the sequence $\{X_n\}$ is stochastically convergent to zero iff the sequence $\{F_n(x)\}$ satisfies the relation $\lim_{n \rightarrow \infty} F_n(x) = \begin{cases} 0 & \text{for } x < 0, \\ 1 & \text{for } x > 0. \end{cases}$		
4	14.a.	Examine that, A process stationary in the wide sense is continuous iff its covariance function $R(\tau)$ is continuous at zero.	K4	CO4
	(OR)			
	14.b.	Analyze the statement "Let $\{X_t, 0 \leq t < \infty\}$ be a homogeneous markov process with a finite number of states $i = 0, 1, 2, \dots, s$. If there exists a $t^* (0 \leq t^* < \infty)$ such that $P_{ij}(t^*) > 0$ ($i, j = 0, 1, \dots, s$), then the limits $\lim_{t \rightarrow \infty} p_{ij}(t) = p_j > 0$ ($i, j = 0, 1, \dots, s$) exist".		
5	15.a.	Prove that - The sequence $\{F_n(t)\}$ of distribution functions of student's t with n degrees of freedom satisfies for every t the relation $\lim_{n \rightarrow \infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-t^2/2} dt.$	K5	CO5
	(OR)			
	15.b.	From a population in which the characteristic X has the normal distribution $N(1; 2)$, we draw a simple sample of size $n = 12$. We observe the following values of X : $x_1 = 2.0, x_2 = 2.5, x_3 = 0.5, x_4 = 1.0, x_5 = 0.0, x_6 = -0.9, x_7 = 5.1, x_8 = -1.5, x_9 = 0.8, x_{10} = 1.1, x_{11} = 0.8, x_{12} = 0.4$. Estimate the probability that Z will exceed or equal the value obtained, = 32.28 ?		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	The characteristic function of the random variable X is given by the formula $\phi(t) = \exp(-\frac{t^2}{2})$. Find the density function of this random variable.	K1	CO1
2	17	Denote by V the velocity of a particle of an ideal gas. Consider the rectangular system of co-ordinates x_1, x_2, x_3 . Let $\vec{V}$ be the velocity vector of the particle, & V_1, V_2, V_3 its projections on the corresponding axes. The position & length of the vector $\vec{V}$ have a random character; hence (V_1, V_2, V_3) is a three-dimensional random variable. Determine its density function.	K5	CO2
3	18	State and prove De moivre-Laplace theorem.	K3	CO3
4	19	Analyze the statement "A function $R(\tau)$ is the correlation function of a process $\{Z_t, -\infty < t < +\infty\}$ stationary in the wide sense, continuous, & satisfying $m = 0, \sigma = 1$ iff there exists a distribution function $F(\lambda)$ such that $R(\tau) = \int_{-\infty}^{+\infty} e^{i\lambda\tau} dF(\lambda)$ ".	K4	CO4
5	20	Derive the Snedecor's F test from Fisher's Z test.	K5	CO5

Z-Z-Z END