

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – MATHEMATICS

MATHEMATICAL METHODS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. If $D(\lambda) \neq 0$, then an integral equation has ----- solutions.
a) only one b) two c) many d) zero
2. The inhomogeneous Fredholm Integral equation with separable kernel has ----- solution.
a) zero b) one
c) two d) infinite
3. An initial value problems in ordinary differential equations leads to a
a) Volterra I.E of second kind b) Fredholm I.E of second kind
c) Volterra I.E of first kind d) Fredholm I.E of first kind
4. If F is independent of y' , then the Euler equation has the form-----
a) $F_y(x,y) = 0$ b) $F_x(x,y) = 0$ c) $F_{y'} = 0$ d) $F_{y'y'} = 0$
5. If the pencil of curves intersect at a common point, then it forms a
(a) field (b) central field
(c) proper field (d) none of these

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6a) Show that the integral equation $g(s) = f(s) + \frac{1}{\pi} \int_0^{2\pi} (\sin(s+t))g(t)dt$ possesses no solution for $f(s) = s$, but that it possesses infinitely many solutions when $f(s) = 1$.
OR
- b) Find the resolvent kernel for the Integral equation $g(s) = f(s) + \int_{-1}^1 (st + s^2 t^2)g(t)dt$
- 7 a) Find the Neumann series for the solution of the integral equation.
 $g(s) = (1+s) + \lambda \int_0^1 (s-t)g(t)dt$.
OR
- b) Find the eigen values and eigen functions of the homogeneous integral equation
 $g(s) = \lambda \int_0^1 \left(st + \frac{1}{st}\right)f(t)dt$.
- 8 a) Convert an Initial value problem to the volterra integral equation of II kind.
OR
- b) Reduce the initial value problem $y''(s) + \lambda y(s) = F(s)$, $y(0)=1$, $y'(0)=0$ to volterra integral equation.

Cont....

- 9 a) On what curves can the functional $v(y(x)) = \int_0^1 (y'^2 + 12xy) dx$ $y(0)=0, y(1)=1$ be extremized?

OR

- b) State and prove the fundamental lemma of the calculus of variations.

- 10 a) Test for an extremum the functional $v(y(x)) = \int_0^a y'^3 dx$, $y(0)=0, y(a)=b, a>0, b>0$.

OR

- b) Test for an extremum of the functional $v(y(x)) = \int_0^{x_1} \left(\frac{\sqrt{1+y'^2}}{\sqrt{y}} \right) dx$, $y(0)=0, y(x_1)=y_1$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11a) Solve the Fredholm Integral equation of the second kind $g(s) = \lambda + \int_0^1 (s+t)g(t)dt$

OR

- b) Solve the integral equations using Approximate method

$$G(s) = e^s - s - \int_0^1 s(e^{st} - 1)g(t)dt.$$

- 12 a) State and prove Fredholm's First theorem on Integral equation.

OR

- b) Solve the integral equation $g(s) = s + \lambda \int_0^1 (st + \sqrt{st})g(t)dt$ using Fredholm's first theorem.

- 13 a) Convert a boundary value problem to the Fredholm integral equation of II kind

OR

- b) Solve the integral equation $f(s) = \int_0^1 \frac{g(t)}{(\cos t - \cos s)^{1/2}} dt$.

- 14 a) Derive Euler Equation of the functional $F(x, y, y')$.

OR

- b) Prove that the arc length of the curve $I[y(x)] = \int_{x_0}^{x_1} \sqrt{1+y'^2} dx$ has extremals are straight lines $y=c_1 x+c_2$.

- 15 a) Test for an extremum the functional $v(y(x)) = \int_0^a \frac{y}{y'^2} dx, y(0)=1, y(a)=b, a>0, 0<b<1$.

OR

- b) Is Jacobi condition fulfilled for the extremal of the functional

$$v(y(x)) = \int_0^{\pi/2} (y'^2 + y^2 + x^2) dx \text{ that passes through the point } A(0,0) \text{ and } B(a,0).$$