

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024

(Third Semester)

Branch - MATHEMATICS

MAJOR ELECTIVE COURSE – II STOCHASTIC DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	An optimal stopping problem can also be expressed in terms of a set of ____ a) variational inequalities b) distributions c) expected profit d) none	K1	CO1
	2	The Mathematical model for a random quantity is a ____ a) σ - algebra b) measurable set c) random variable d) Probability measure	K2	CO1
2	3	Itô integral is defined by ____ a) $I[f](w) = \int_S^T f(t, w) dB_t(w)$ b) $I[f](w) = \int_S^T f(t, w) dB(w)$ c) $I[f] = \int_S^T f(t, w) dB_t(w)$ d) $I[f](w) = \int_T^S f(t, w) dB_t(w)$	K1	CO2
	4	$N_t = B_t^3 - 3t B_t$ is a ____ a) Brownian motion b) Martingale c) iterated Itô integrals d) none	K2	CO2
3	5	The solution X_t found above is called a ____ solution, because the variation B_t of Brownian motion is given in advance and the solution X_t constructed from it is Ft^z - adapted. a) Weak b) General c) Strong d) Classical	K1	CO3
	6	Itô processes is also called the ____ a) Brownian motion b) Stochastic process c) Stochastic integrals d) none	K2	CO3
4	7	An expression for X_t by solving the (linear) stochastic differential equation ____ a) $dX_t = F(t)X_t dt + C(t)dU_t$ b) $dX_t = F(t)X_t dt + C(t)dt$ c) $dX_t = F(t)dt + C(t)dU_t$ d) $dX_t = F(t)X_t dt + C(t)dU$	K1	CO4
	8	For the solution of linear filtering problems governed by more general processes then ____ a) dimension on the motion b) invertible c) Brownian motion d) asymptotic estimation	K2	CO4
5	9	The harmonic measure of X on $\partial G, \mu_G^x$, is ____ a) $\mu_G^x(F) = Q^x[X_{\tau_G} \in F]$ b) $\mu_G^x(F) = [X_{\tau_G} \in F]$ c) $\mu_G^x(F) = Q^x[X_G \in F]$ d) $\mu_G^x(Q) = F[X_{\tau_G} \in F]$	K1	CO5
	10	If $Z(t)$ is a lower bounded local martingale, then $Z(t)$ is a ____ a) local martingale b) uniformly integrals c) super martingale d) N_t -martingale	K2	CO5

Cont...

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove Kolmogorov's continuity theorem.	K3	CO1
		(OR)		
	11.b.	If $X, Y: \Omega \rightarrow R^n$ are two given functions, then prove that Y is H_X -measurable if and only if there exists a Borel measurable function $g: R^n \rightarrow R^n$ such that $Y = g(X)$.		
2	12.a.	Let $f \in v(0, T)$. Then prove that there exists a t -continuous version of $\int_0^t f(s, w) dB_s(w)$; $0 \leq t \leq T$, i.e. there exists a t -continuous stochastic process J_t on $(\Omega, \mathcal{F}, P)$ such that $P \left[J_t = \int_0^t f dB \right] = 1$ for all $t, 0 \leq t \leq T$.	K2	CO2
		(OR)		
	12.b.	Write a comparison of Itô and Stratonovich integrals.		
3	13.a.	State and prove the Itô representation theorem.	K3	CO3
		(OR)		
	13.b.	State and prove existence and uniqueness theorem for stochastic differential equations.		
4	14.a.	Prove that $M_t = \begin{bmatrix} X_t \\ Z_t \end{bmatrix} \in R^2$ is a Gaussian process.	K4	CO4
		(OR)		
	14.b.	Let $X, Z_s; s \leq t$ be random variables in $L^2(P)$ and assume that $X, Z_{s_1}, Z_{s_2}, \dots, Z_{s_n} \in R^{n+1}$ has a normal distribution for all $s_1, s_2, \dots, s_n \leq t, n \geq 1$. Then prove that $\mathcal{P}_L(X) = E[X G] = \mathcal{P}_K(X)$.		
5	15.a.	If x is not a trap for X_t , then prove that there exists an open set $U \ni x$ such that $E^x[J_U] < \infty$.	K4	CO5
		(OR)		
	15.b.	State and prove the theorem on strong Markov property for Itô diffusions.		

SECTION -C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Explain Brownian motion with an example.	K4	CO1
2	17	State and prove the Itô isometry lemma.	K4	CO2
3	18	State and prove the theorem on Integration by parts.	K4	CO3
4	19	Prove that i) N_t has orthogonal increments ii) $E[N_t^2] = \int_0^t D^2(s) ds$ iii) $\mathcal{L}(N, t) = \mathcal{L}(Z, t)$ for all $t \geq 0$ iv) N_t is a Gaussian process	K4	CO4
5	20	State and prove the theorem on Markov property for Itô diffusions.	K4	CO5

Z-Z-Z

END