

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch – MATHEMATICS

FUNCTIONAL ANALYSIS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The null space of T is the set of all $x \in D(T)$ such that _____. a) $Tx \neq 0$ b) $Tx = 0$ c) $T(\alpha x) = \alpha T(x)$ d) $T0 = 0$	K1	CO1
	2	If a normed space has a Schauder basis then it is _____. a) separable b) absolutely convergent c) schauder basis d) convex set	K2	CO1
2	3	If $x \perp y$ in an inner product space X then $\ x + y\ ^2 =$ _____. a) $\ x\ ^2$ b) $\ y\ ^2$ c) $\ x - y\ ^2$ d) $\ xy\ ^2$	K1	CO2
	4	If $x(t) = \begin{cases} t & \text{if } -\pi/2 \leq t < \pi/2 \\ \pi - t & \text{if } \pi/2 \leq t < 3\pi/2 \end{cases}$, then $a_K =$ _____. a) 0 b) 1 c) 2π d) $4/\pi$	K2	CO2
3	5	A chain is a partially ordered set such that every two elements of the set are _____. a) incomparable b) comparable c) partially ordered d) totally ordered	K1	CO3
	6	A maximal element of M is an $m \in M$ such that $m \leq x$ implies _____. a) mx b) $m \neq x$ c) $m = x$ d) $m = 0$	K2	CO3
4	7	If X is _____ with a subspace Z then X is said to be embeddable in a normed space. a) isomorphic b) homeomorphic c) homomorphic d) unique fixed point	K1	CO4
	8	A vector space X is algebraically reflexive if the canonical mapping $C: X \rightarrow X^{**}$ is _____. a) injective b) bijective c) surjective d) reflexive	K2	CO4
5	9	A mapping $T: X \rightarrow X$ is a contraction on X if there is a positive real number $\alpha < 1$ such that for all $x, y \in X$ then _____. a) $d(Tx, Ty) < \alpha d(x, y)$ b) $d(Tx, Ty) \leq \alpha d(x, y)$ c) $d(Tx, Ty) = d(x, y)$ d) $d(Tx, Ty) \geq \alpha d(x, y)$	K1	CO4
	10	If there is a constant K such that for all $x, y \in [a, b]$, $ Tx - Ty \leq K x - y $ then the mapping $T: [a, b] \rightarrow [a, b]$ is said to satisfy a _____. a) Fixed point b) Contraction c) Libschitz condition d) Decreasing map	K2	CO4

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that a subspace Y of a Banach Space X is complete if and only if the set Y is closed in X .	K2	CO1
		(OR)		
	11.b.	If a normed space X is finite dimensional then Prove that every linear operator on X is bounded.		
2	12.a.	State and Prove Schwarz inequality.	K3	CO2
		(OR)		
	12.b.	Let Y be any closed subspace of a Hilbert space H then $H = Y \oplus Z$, $Z = Y^\perp$.		
3	13.a.	State and Prove the Riesz representation theorem.	K3	CO3
		(OR)		
	13.b.	Prove that an every Hilbert Space $H \neq \{0\}$ there exists a total orthonormal set.		
4	14.a.	If the dual space X' of a normed space X is separable then prove that X itself is separable.	K4	CO4
		(OR)		
	14.b.	State and Prove Uniform boundedness theorem.		
5	15.a.	Prove that the range $\mathcal{R}(T)$ of a compact linear operator $T: X \rightarrow Y$ is separable if X & Y are normed spaces.	K4	CO4
		(OR)		
	15.b.	Let (T_n) be a sequence of compact linear operator from a normed space X into a Banach space Y . If (T_n) is uniformly operator convergent, say $\ T_n - T\ \rightarrow 0$ then the limit operator T is compact.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that an every finite dimensional subspace Y of a normed space X is complete. In particular every finite dimensional normed space is complete.	K4	CO1
2	17	State and Prove Bessel's Inequality.	K4	CO2
3	18	State & prove Hahn Banach Theorem.	K4	CO3
4	19	Prove that a bounded linear operator T from a Banach space X on a Banach space Y is an open mapping. Hence if T is bijective, T^{-1} is continuous and then bounded.	K4	CO4
5	20	Let $T: X \rightarrow Y$ be a compact linear operator on a normed space X then for every $\lambda \neq 0$, the range of $T_\lambda = T - \lambda I$ is closed.	K4	CO4

Z-Z-Z

END