

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - MATHEMATICS

DIFFERENTIAL GEOMETRY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The binomial line at P is the normal in a direction orthogonal to the _____. a) torsion b) osculating plane c) curvature d) evolute	K1	CO1
	2	The involutes of a circular helix are _____. a) curvature b) circle c) sphere d) plane curves	K2	CO1
2	3	The parametric longitude curves $v=\text{constant}$. a) Meridians b) Parallels c) Revolution d) Helicoid	K1	CO1
	4	One period of a _____ of pitch $2\pi a$ corresponds isometrically to a whole catenoid of parameter. a) Helicoid b) Right helicoid c) Curvature d) Torson	K2	CO1
3	5	Every helix on a cylinder is a _____. a) Curvature b) Geodesic c) Catenoid d) Meridians	K1	CO1
	6	Two surfaces of the same constant curvature are _____. a) Isometric b) Locally isometric c) Orthogonal d) Parallel	K2	CO1
4	7	If ϕ denotes the angle between the principal normal $\vec{n}$ to a curve on the surface and the surface normal $\vec{N}$ then $k\cos\phi = K_n$. This is known as a) Rodrigue's theorem b) Euler theorem c) Meusnier theorem d) Dupin indication theorem	K1	CO1
	8	_____ contains all three types of points. a) Catenoid b) Helicoids c) Bertrand curves d) Anchor ring	K2	CO1
5	9	The only compact surface of class ≥ 2 for which every point is an umbilic are _____. a) Spheres b) Helices c) Anchor ring d) Circles	K1	CO1
	10	In a closed region R of a surface of constant positive _____ curvature without umbilicus, the principal curvatures take their extreme values at the boundary. a) Gaussian b) Principal c) Mean d) Geodesic	K2	CO1

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the length of the curve given as the intersection of the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a \cosh(z/a)$ from the point $(a, 0, 0)$ to the point (x, y, z) .	K3	CO2
		(OR)		
	11.b.	State and prove Serret-Frenet Formulae.		
2	12.a.	Derive the angle between parametric curves.	K3	CO2
		(OR)		
	12.b.	On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.		
3	13.a.	A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.	K3	CO2
		(OR)		
	13.b.	Prove that if the orthogonal trajectories of the curves $v = \text{constant}$ are geodesics, and then $\frac{H^2}{E}$ is independent of u .		
4	14.a.	State and prove Meusnier's theorem.	K3	CO3
		(OR)		
	14.b.	Prove that the edge of regression of the rectifying developable has equation $\bar{R} = \bar{r} + \kappa \frac{(\bar{\pi} + \kappa \bar{b})}{(\kappa' \tau - \kappa \tau')}$		
5	15.a.	Prove that the only compact surfaces whose Gaussian curvature is positive and mean curvature constant are spheres.	K3	CO3
		(OR)		
	15.b.	Explain complete surfaces.		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that the radius of the curvature of the locus of the Centre of curvature of a curve is given by $\left[\left\{ \frac{\rho^2 \sigma}{R^3} \frac{d}{ds} \left(\frac{\sigma \rho'}{\rho} \right) - \frac{1}{R} \right\}^2 + \frac{\rho'^2 \sigma^4}{\rho^2 R^4} \right]^{-1/2}$	K4	CO4
2	17	Find a surface of revolution which is isometric with a region of the right helicoid?	K4	CO4
3	18	State and prove Gauss Bonnet theorem.	K4	CO4
4	19	Derive Rodrigues' formula.	K4	CO5
5	20	Prove that the only compact surface of class ≥ 2 for which every point is an umbilic are spheres.	K4	CO5

Z-Z-Z

END