

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch- MATHEMATICS

TOPOLOGY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The collection consisting of X and ϕ only is also a topology on X which is called Topology. (a) Standard (b) Discrete (c) Indiscrete (d) Product	K1	CO1
	2	Let X be a topological space and A is a subset of X . If the union of all open sets contained in A then it is called (a) Closure of A (b) Interior of A (c) Closure of X (d) Open in A	K2	CO1
2	3	Let δ_β denote the collection $\delta_\beta = \{\pi_\beta^{-1}(U_\beta)/U_\beta \text{ open in } X_\beta\}$ and δ denote the union of these collections $\delta = \bigcup_{\beta \in J} \delta_\beta$ the topology generated by the sub basis δ is called (a) Standard topology (b) Product topology (c) Discrete topology (d) Indiscrete topology	K1	CO2
	4	Let X is a topological space. If there exists a metric d on the set X that induces the topology of X then X is said to be (a) Metrizable (b) Bounded (c) Basis (d) Open	K2	CO2
3	5	The union of a collection of connected subspaces of X that have a point in common is (a) connected (b) Continuous (c) Closed (d) Bounded	K1	CO3
	6	A space X is locally connected if and only if for every open set U of X each component of U is (a) Closed in X (b) Bounded in X (c) Open in X (d) Connected in X	K2	CO3
4	7	Let (X, d) be a metric space. If f satisfies the condition $d(f(x), f(y)) < d(x, y)$ for all $x, y \in X$ with $x \neq y$ then f is called (a) Shrinking map (b) Fixed point (c) Contraction (d) Compactness	K1	CO4
	8	A sub set A of a space X is said to be dense in X if (a) $\bar{A} = X$ (b) $A = X$ (c) $\bar{A} \neq X$ (d) $A \neq X$	K2	CO4
5	9	Every compact Hausdroff space is (a) Compact (b) Connected (c) Bounded (d) Normal	K1	CO5
	10	Every regular space X with a countable basis is (a) Countable (b) Connected (c) Metrizable (d) Compact	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies $\mathcal{T}$ and $\mathcal{T}'$ respectively on X then prove that the following are equivalent (i) $\mathcal{T}'$ is finer than $\mathcal{T}$ (ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$	K4	CO1
		(OR)		

Cont...

Q.No 11 Cont....

1	11.b.	If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y then show that the collection $\mathcal{D} = \{B \times C \mid B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$?	K4	CO1
2	12.a.	Let X be a metric space with metric d . Define $\bar{d}: X \times X \rightarrow \mathbb{R}$ by the equation $\bar{d}(x, y) = \min\{d(x, y), 1\}$ then prove that $\bar{d}$ is a metric that induces the same topology as d ?	K3	CO2
	(OR)			
	12.b.	Let $f: X \rightarrow Y$. If the function f is continuous then examine that for every convergent sequence $x_n \rightarrow x$ in X the sequence $f(x_n)$ converges to $f(x)$. The converse holds if X is metrizable?		
3	13.a.	Prove that the image of a connected space under a continuous map is connected?	K3	CO3
	(OR)			
	13.b.	Let X be a topological space. Then examine whether X is compact if and only if for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property the intersection $\bigcap_{C \in \mathcal{C}} C$ of all the elements of $\mathcal{C}$ is nonempty?		
4	14.a.	Suppose that X has a countable basis then verify the following (i) Every open covering of X contains a countable subcollection covering X (ii) There exists a countable subset of X that is dense in X ?	K4	CO4
	(OR)			
	14.b.	Verify that a subspace of a Hausdroff space is Hausdroff and product of Hausdroff space is Hausdroff?		
5	15.a.	Prove that every compact Hausdroff space is normal?	K5	CO5
	(OR)			
	15.b.	State and prove Tychonoff theorem?		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let X be a topological space then show that the following conditions hold (i) $\emptyset$ and X are closed (ii) Arbitrary intersection of closed sets are closed (iii) Finite unions of closed sets are closed	K4	CO1
2	17	Show that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are same as the product topology on $\mathbb{R}^n$?	K5	CO2
3	18	Prove that a finite Cartesian product of connected space is connected?	K5	CO3
4	19	If X be a metrizable space then verify the following (i) X is compact (ii) X is limit point compact (iii) X is sequentially compact	K4	CO4
5	20	Prove that every well ordered set X is normal in the order topology?	K4	CO5

Z-Z-Z

END