

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch- **MATHEMATICS**

OPTIMIZATION TECHNIQUES

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	An arc is said to be ----- if it allows positive flow in one direction. a) directed b) undirected c) path d) oriented	K1	CO1
	2	A path forms a ----- if it connects a node to itself through other nodes. a) activity b) loop c) dummy d) cycle	K2	CO1
2	3	In the DP terminology, x_i is referred to as the ---- of the system at stage i. a) state b) system c) finite d) infinite	K1	CO2
	4	Which one of the following is represented as the state at any stage is a single element? a) weight b) volume c) mass d) height	K2	CO2
3	5	The -----, a modeling scheme that estimates stochastic parameters based on random samples. a) Dijkstra's b) Monte Carlo technique c) Floyd's d) none of these	K1	CO3
	6	----- models deal with systems whose behaviour changes continuously with time. a) continuous b) jump c) discontinuous d) discrete	K2	CO3
4	7	An ----- of a function $f(X)$ defines either a maximum or a minimum of the function. a) extreme point b) saddle c) inflation d) local	K1	CO4
	8	A point x_0 is -----, if $f(X_0 + h) \leq f(X_0)$. a) minimum b) local c) global d) maximum	K2	CO4
5	9	Which method is converges fast in direct search method? a) dichotomous b) Dijkstra's c) golden section d) Floyd's	K1	CO5
	10	Termination of the gradient method occurs at the point where the gradient vector becomes -----. a) null b) zero c) one d) infinite	K2	CO5

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SECTION - B (35 Marks)

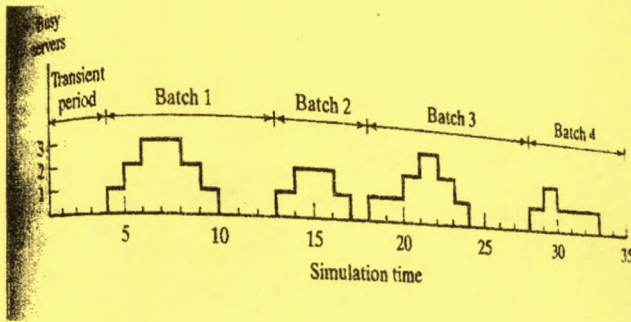
Answer ALL questions

ALL questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	The network in the following figure gives the permissible routes and their lengths in miles between city 1 and four other cities. Determine the shortest route from node 1 to node 2.	K6	CO1
		(OR)		
	11.b.	Determine the critical path for the project network in the following figure. Given the durations are in days.		
2	12.a.	For the network in the below, it is desired to determine the shortest route between cities 1 to 7. Define the stages and the states using backward recursion, and then solve the problem.	K5	CO2
		(OR)		
	12.b.	A construction contractor estimates that the size of the work force needed over the next 5 weeks to be 5, 7, 8, 4 and 6 workers respectively. Excess labour kept on the force will cost \$300 per worker per week, and new hiring in any week will incur a fixed cost of \$400 plus \$200 per worker per week.		

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3	13.a.	Generate three random numbers based on the multiplicative congruential method using $b = 9, c = 5, m = 12$ and $seed = 11$.	K6	CO3
	(OR)			
	13.b.	The following figure represents the number of busy servers in a single facility with three parallel servers. The length of the simulation run is 35 time units and the length of the transient period is 4 time units. Estimate the average utilization of the facility based on the regenerative method. 		
4	14.a.	Show that a necessary condition for X_0 to be an extreme point of $f(X)$ is that $\nabla f(X_0) = 0$.	K5	CO4
	(OR)			
	14.b.	Solve by the following by Lagrangean method; $Minimize f(X) = x_1^2 + x_2^2 + x_3^2$ Subject to $g_1(X) = x_1 + x_2 + 3x_3 - 2 = 0$; $g_2(X) = 5x_1 + 2x_2 + x_3 - 5 = 0$		
5	15.a.	Solve: $Maximize Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ Subject to $X_1 + 2X_2 \leq 2$ $X_1, X_2 \geq 0$	K5	CO5
	(OR)			
	15.b.	Convert the following stochastic problem into an equivalent deterministic model: $Max z = 5x_1 + 6x_2 + 3x_3$ Subject to $P\{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq 8\} \geq 0.95$ $P\{5x_1 + x_2 + 6x_3 \leq b_2\} = 0.10$ $x_1, x_2, x_3 \geq 0$ Assume that the parameters $a_{ij}, j = 1, 2, 3$, are independent and normally distributed random variables with the following means and variances; $E\{a_{11}\} = 1, E\{a_{12}\} = 3, E\{a_{13}\} = 9$ $var\{a_{11}\} = 25, var\{a_{12}\} = 16, var\{a_{13}\} = 4$ The parameters b_2 is normally distributed with mean 7 and variance 9.		

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SECTION -C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Midwest TV cable company is in the process of providing cable service to five new housing development areas. Figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network.	K6	CO1
2	17	Suppose that you want to invest \$4000 now and \$2000 at the start of years 2 to 4. The interest rate offered by First bank is 8% compounded annually, and the bonuses over the next 4 years are 1.8%, 1.7%, 2.1% and 2.5% respectively. The annual interest rate offered by second bank is .2% lower than that of first bank, but its bonus is .5% higher. Determine the objective is to maximize the accumulated capital at the end of 4 years.	K5	CO2
3	18	Describe about the manual simulation of a single server model. Explain (i) arrival event, (ii) departure event	K6	CO3
4	19	Solve by Newton Raphson method $g(x) = (3x - 2)^2(2x - 3)^2$	K5	CO4
5	20	Approximate the following ; $Maximize z = x_1 + x_2^4$ Subject to $3x_1 + 2x_2^2 \leq 9; x_1, x_2 \geq 0.$	K5	CO5

Z-Z-Z END

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Branch - MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

Q. No.	SECTION A (10 x 1 = 10) Answer ALL Questions	Level	CO
1.	The equation $x'' - 2tx' + 2nx = 0$ is known as (a) Bessel equation (b) Hermite equation (c) Legendre equation (d) None of the above	K2	CO1
2.	The Bessel equation possesses a regular singular point at — (a) 0 (b) 1 (c) -1 (d) 2	K1	CO1
3.	A solution matic ϕ of $x' = A(t)x, t \in I$ on I is a fundamental matrix of IVP on I iff (a) $\det\phi(t) = 0, t \in I$ (b) $\det\phi(t) = 1, t \in I$ (c) $\det\phi(t) \neq 0, t \in I$ (d) $\det\phi(t) \neq 1, t \in I$	K1	CO2
4.	The Solution of the IVP $x'' - 2x_0' + x = 0; x(0) = 0, x'(0) = 1$ is — (a) $x(t) = e^t, t \in I$ (b) $x(t) = e^{-t}, t \in I$ (c) $x(t) = -te^{-t}, t \in I$ (d) $x(t) = te^t, t \in I$	K2	CO2
5.	Let ϕ denote fundamental matrix for $x' = A(t)x$ Then $\phi(t + \omega), (-\infty < t < \infty)$ is — (a) Fundamental matrix (b) Exporential matrix (c) Square matrix (d) None of the above	K1	CO3
6.	Consider the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then e^{At} is (a) $\begin{bmatrix} \cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$ (b) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (c) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (d) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$	K2	CO3
7.	A function $f(t, x)$ satisfies Lipschitz condition if their exists a constant $k > 0, (t, x_1), (t, x_2) \in D$ and $x_1 \neq x_2$, then (a) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } \leq K$ (b) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } < K$ (c) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } \geq K$ (d) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } > K$	K1	CO4

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8.	Consider the IVP $x' = x^2, x(0) = 1$. The equation is equivalent to (a) $x(t) = 1 - \int_{t_0}^t x^2(s)ds$ (b) $x(t) = 1 + \int_{t_0}^t x^2(s)ds$ (c) $x(t) = 1 - \int_{t_0}^t x(s)ds$ (d) $x(t) = 1 + \int_{t_0}^t x(s)ds$	K2	CO4
9.	A linear BVP which is not regular is called — (a) Singular non linear BVP (b) Singular linear BVP (c) Singular non linear IVP (d) Singular linear IVP	K1	CO5
10.	Two distinct functions x and y defined and continuous on $[A, B]$ are said to be orthogonal with respect to a continuous weight function $r(t)$ if (a) $\int_A^B r(s)x(s)y(s)ds \neq 0$ (b) $\int_A^B r(s)x(s)y(s)ds = 0$ (c) $\int_A^B r(s)x(s)y(s)ds = 1$ (d) $\int_A^B r(s)x(s)y(s)ds \neq 1$	K2	CO5

	SECTION B (5 x 7 = 35) (Answer ALL questions)		
11(a).	If $P_n(t)$ and $P_m(t)$ are Legendre Polynomials. Then Prove that $\int_{-1}^1 P_n(t)P_m(t)dt = 0$ if $m \neq n$. OR	K2	CO1
11(b).	Find the regular singular point in the equation $t(t-1)^2(t+3)x'' + t^2x' - (t^2+t-1)x = 0$.	K2	CO1
12(a).	Let $A(t)$ be an $n \times n$ matrix which is continuous on I . Suppose a matrix ϕ satisfies $x' = A(t)x, \quad t \in I$ Then prove that $\det \phi$ satisfies the first order equation $(\det \phi)' = (\text{tr } A)(\det \phi)$. OR	K3	CO2
12(b).	Find the fundamental system of solutions for the system $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; t = [0, 1]$.	K3	CO2
13(a).	Find the fundamental matrix of the linear system $x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix} x$. OR	K2	CO3
13(b).	Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.	K2	CO3

14(a).	Consider the IVP $x' = x^2 + \cos^2 t$, $x(0) = 0$. To determine the largest interval of existence of its solution.	K4	CO4
OR			
14(b).	Prove that the error $x(t) - x_n(t)$ satisfies the estimate $ x(t) - x_n(t) \leq \frac{L(Kh)^{n+1}}{K(n+1)!} e^{Kh}; \quad t \in [t_0, t_0 + h]$	K4	CO4
15(a).	Using appropriate Green's functions. Solve $x'' = e^t$, $x(0) = x(1) = 0$.	K5	CO5
OR			
15(b).	consider the equation $x'' + \lambda x = 0$, $0 \leq t \leq \pi$. Find the eigenvalues and eigenfunctions in the following cases: (i) $x'(0) = x'(\pi) = 0$ (ii) $x(0) = 0, x'(\pi) = 0$	K5	CO5

SECTION -C (3 × 10=30) (Answer ANY THREE questions)			
16.	Derive the solutions of Bessel Equation.	K3	CO1
17.	State and prove Existence and Uniqueness theorem.	K4	CO2
18.	Determine e^{tA} for the system $x' = Ax$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$.	K2	CO3
19.	State and prove Picard's theorem.	K2	CO4
20.	Prove that $G(t; s) = \begin{cases} -y(t)z(s)/A & \text{if } t \leq S \\ -y(s)z(t)/A & \text{if } t \geq S \end{cases}$ then $x(t)$ is a solution of $L(x) + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(\epsilon, s) f(s) ds$	K5	CO5

Z-Z-Z

END