

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch - MATHEMATICS

MEASURE THEORY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$m^*(A) \geq$ i) 0 ii) ∞ iii) 1 iv) A	K1	CO1
	2	If $F \in \mathcal{M}$ and ____ then G is measurable. i) $m^*(F \Delta G) = F$ ii) $m^*(F \Delta G) = G$ iii) $m^*(F \Delta G) = 1$ iv) $m^*(F \Delta G) = 0$	K2	CO1
2	3	If f is a non-negative measurable function, then $f = 0$ a. e if and only if $\int f dx =$ i) x ii) 1 iii) f iv) 0	K1	CO2
	4	If f is a measurable function such that at least one of $\int f^+ dx$, $\int f^- dx$ is finite then, $\int f dx =$ i) $\int f^+ dx - \int f^- dx$ ii) $\int f^+ dx - \int f^- dx$ iii) $\int f^+ dx + \int f^- dx$ iv) $\int f^- dx - \int f^+ dx$	K2	CO2
3	5	A class of sets $\mathcal{R}$ of some fixed space is called a ____ if whenever $E \in \mathcal{R}$ then $E \cup F$ and $E - F$ belongs to $\mathcal{R}$ i) integral domain ii) Group iii) Ring iv) Field	K1	CO3
	6	A pair $[X, \mathcal{S}]$ where $\mathcal{S}$ is a σ -algebra of subsets of a space X , is called a ____. i) measurable space ii) measure space iii) metric space iv) Euclidean space	K2	CO3
4	7	$L^\infty(X, \mu)$ is a ____ over the real numbers i) metric space ii) vector space iii) Euclidean space iv) measurable space	K1	CO4
	8	$\ f + g\ _\infty \leq$ i) $\ f\ _\infty$ ii) $\ g\ _\infty$ iii) $\ f\ _\infty + \ g\ _\infty$ iv) $\ f\ _\infty - \ g\ _\infty$	K2	CO4
5	9	If $\phi(E) = \int_E f d\mu$ where $\int f d\mu$ is defined then ϕ is a ____ i) signed measure ii) unsigned measure iii) relative measure iv) no measure	K1	CO5
	10	If μ is a measure, $\int f d\mu$ exists and $v(E) = \int_E f d\mu$ then ____ i) $v \gg \mu$ ii) $v \ll \mu$ iii) $v^+ \ll \mu$ iv) $v^+ \gg \mu$	K2	CO5

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	For any sequence of sets $\{E_i\}$, examine $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$	K4	CO1
		(OR)		
	11.b.	Let f be a measurable function and B a Borel set; then inspect $f^{-1}(B)$ is a measurable set.		
2	12.a.	Explain: Let f be a non-negative measurable function. Then there exists a sequence $\{\varphi_n\}$ of measurable simple functions such that for each x , $\varphi_n(x) \uparrow f(x)$	K2	CO2
		(OR)		
	12.b.	State and Prove Lebesgue's Dominated Convergence Theorem		
3	13.a.	Examine: Let $\{A_i\}$ be a sequence in a ring $\mathcal{R}$ then there is a sequence $\{B_i\}$ of disjoint sets of $\mathcal{R}$ such that $B_i \subseteq A_i$ for each i and $\bigcup_{i=1}^N A_i = \bigcup_{i=1}^N B_i$ for each N , so that $\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} B_i$	K4	CO3
		(OR)		
	13.b.	Inspect: If μ is a measure on a σ -ring $\mathcal{S}$, then the class $\bar{\mathcal{S}}$ of sets of the form $E \Delta N$ for any sets E, N such that $E \in \mathcal{S}$ while N is contained in some set in $\mathcal{S}$ of measure zero is a σ -ring, and the set function $\bar{\mu}$ defined by $\bar{\mu}(E \Delta N) = \mu(E)$ is complete measure on $\bar{\mathcal{S}}$.		
4	14.a.	Explain: Let $f, g \in L^p(\mu)$ and let a, b be constants; then $af + bg \in L^p(\mu)$	K2	CO4
		(OR)		
	14.b.	State and Prove Jensen's Inequality		
5	15.a.	Build: A countable union of sets positive with respect to a signed measure ν is a positive set.	K3	CO5
		(OR)		
	15.b.	Build: Let ν be a signed measure on $[X, \mathcal{S}]$. Then there exist measures ν^+ and ν^- on $[X, \mathcal{S}]$ such that $\nu = \nu^+ - \nu^-$ and $\nu^+ \perp \nu^-$. The measures ν^+ and ν^- are uniquely defined by ν and $\nu = \nu^+ - \nu^-$ is said to be the Jordan decomposition of ν		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Examine: The outer measure of an interval equals its length.	K4	CO1
2	17	State and Prove Fatou's Lemma	K5	CO2
3	18	Explain: Let μ^* be an outer measure on $\mathcal{K}(\mathcal{R})$ and let $\mathcal{S}^*$ denote the class of μ^* measurable sets. Then $\mathcal{S}^*$ is a σ ring and μ^* restricted to $\mathcal{S}^*$ is complete measure.	K5	CO3
4	19	Explain: A differentiable function ψ is convex on (a, b) if and only if ψ' is a monotone increasing function. If ψ'' exists on (a, b) then ψ is convex if, and only if $\psi'' \geq 0$ on (a, b) and strictly convex if $\psi'' > 0$ on (a, b)	K5	CO4
5	20	Explain: Let ν be a signed measure on $[X, \mathcal{S}]$. Let $E \in \mathcal{S}$ and $\nu(E) > 0$. Then there exists A , a set positive with respect to ν , such that $A \subseteq E$ and $\nu(A) > 0$	K5	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)
Branch – MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The partial differential equation of $(x - a)^2 + (y - b)^2 + z^2 = 1$ is ____. a) $z(1 + p^2 + q^2) = 1$ b) $z(1 + p^2 + q^2) = 0$ c) $(1 + p^2 + q^2) = z^2$ d) $z^2(1 + p^2 + q^2) = 1$.	K1	CO2
	2	Lagrange's equation is ____. a) $Pdx + Qdy = 0$ b) $Pdy + Qdx = 0$ c) $P_p + Q_q = R$ d) $PQ + pq + R$.	K2	CO1
2	3	Which of the following is wave equation? a) $\frac{\partial^2 \phi}{\partial x^2} = c^2 \frac{\partial^2 \phi}{\partial t^2}$ b) $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$ c) $c^2 \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial t^2}$ d) $\frac{1}{c^2} \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial t^2}$.	K1	CO2
	4	In second order differential equation, what is the condition for parabolic? a) $S^2 - 4RT > 0$ b) $S^2 - 4RT < 0$ c) $S^2 - 4RT \neq 0$ d) $S^2 - 4RT = 0$.	K2	CO2
3	5	If ψ_1 and ψ_2 are solutions of interior Dirichlet problem then $\psi = \psi_1 - \psi_2$ must satisfy ____. a) $\nabla^2 \psi = 0$ b) $\nabla^2 \psi \neq 0$ c) $\psi = 0$ d) $\psi \neq 0$.	K1	CO4
	6	Any problem in which we are required to find such a function ψ is called a BVP for ____. a) Laplace equation b) Poisson equation c) Dirichlet equation d) none of these.	K2	CO5
4	7	The equation $(\nabla^2 + k^2)\psi = 0$ is known as ____ equation. a) Laplace b) telegraphy c) Poisson d) Helmholtz.	K1	CO2
	8	The main technique to find the direct solution of a boundary value problem when the boundary curve Γ has a simple form is ____. a) theory of integral transforms b) method of separation of variables c) Jacobi method d) Charpit method.	K2	CO4
5	9	The general diffusion equation is ____. a) $\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{K}$ b) $\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{K} \frac{\partial \theta}{\partial t}$ c) $\frac{\partial \theta}{\partial x} = K \frac{\partial \theta}{\partial t}$ d) $\nabla^2 \theta = \frac{1}{K} \frac{\partial \theta}{\partial t}$.	K1	CO2
	10	Fink's first law of diffusion is $J = ?$ a) $-C \text{ grad } d$ b) $C \text{ grad } d$ c) $-D \text{ grad } c$ d) $D \text{ grad } c$.	K2	CO2

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the general integrals of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.	K3	CO4
		(OR)		
	11.b.	Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.		
2	12.a.	Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2 \frac{\partial^4 z}{\partial x^2 \partial y^2}$.	K4	CO3
		(OR)		
	12.b.	Find a particular integral of the equation $(D^2 - D')z = e^{2x+y}$.		
3	13.a.	Explain exterior Neumann problem.	K3	CO2
		(OR)		
	13.b.	Find the solution of the one dimensional wave equation by the method of separation of variables.		
4	14.a.	Write a short note on Longitudinal sound waves.	K4	CO2
		(OR)		
	14.b.	Derive the D'Alembert's solution of the one - dimensional wave equation.		
5	15.a.	Derive the elementary solution of Diffusion equation.	K5	CO5
		(OR)		
	15.b.	Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find the complete integral of $z^2 = pqxy$.	K4	CO3
2	17	Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}$ to canonical form and hence solve it.	K5	CO5
3	18	Show that the surfaces $x^2 + y^2 + z^2 = cx^{\frac{2}{3}}$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.	K4	CO4
4	19	Explain several kinds of situations in physics with the occurrence of wave equations.	K5	CO4
5	20	State and prove Duhamel's Theorem.	K4	CO2

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch- MATHEMATICS

TOPOLOGY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The collection consisting of X and ϕ only is also a topology on X which is called Topology. (a) Standard (b) Discrete (c) Indiscrete (d) Product	K1	CO1
	2	Let X be a topological space and A is a subset of X . If the union of all open sets contained in A then it is called (a) Closure of A (b) Interior of A (c) Closure of X (d) Open in A	K2	CO1
2	3	Let δ_β denote the collection $\delta_\beta = \{\pi_\beta^{-1}(U_\beta)/U_\beta \text{ open in } X_\beta\}$ and δ denote the union of these collections $\delta = \bigcup_{\beta \in J} \delta_\beta$ the topology generated by the sub basis δ is called (a) Standard topology (b) Product topology (c) Discrete topology (d) Indiscrete topology	K1	CO2
	4	Let X is a topological space. If there exists a metric d on the set X that induces the topology of X then X is said to be (a) Metrizable (b) Bounded (c) Basis (d) Open	K2	CO2
3	5	The union of a collection of connected subspaces of X that have a point in common is (a) connected (b) Continuous (c) Closed (d) Bounded	K1	CO3
	6	A space X is locally connected if and only if for every open set U of X each component of U is (a) Closed in X (b) Bounded in X (c) Open in X (d) Connected in X	K2	CO3
4	7	Let (X, d) be a metric space. If f satisfies the condition $d(f(x), f(y)) < d(x, y)$ for all $x, y \in X$ with $x \neq y$ then f is called (a) Shrinking map (b) Fixed point (c) Contraction (d) Compactness	K1	CO4
	8	A sub set A of a space X is said to be dense in X if (a) $\bar{A} = X$ (b) $A = X$ (c) $\bar{A} \neq X$ (d) $A \neq X$	K2	CO4
5	9	Every compact Hausdroff space is (a) Compact (b) Connected (c) Bounded (d) Normal	K1	CO5
	10	Every regular space X with a countable basis is (a) Countable (b) Connected (c) Metrizable (d) Compact	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies $\mathcal{T}$ and $\mathcal{T}'$ respectively on X then prove that the following are equivalent (i) $\mathcal{T}'$ is finer than $\mathcal{T}$ (ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$	K4	CO1
		(OR)		

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Q.No 11 Cont....

1	11.b.	If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y then show that the collection $\mathcal{D} = \{B \times C \mid B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$?	K4	CO1
2	12.a.	Let X be a metric space with metric d . Define $\bar{d}: X \times X \rightarrow \mathbb{R}$ by the equation $\bar{d}(x, y) = \min\{d(x, y), 1\}$ then prove that $\bar{d}$ is a metric that induces the same topology as d ?	K3	CO2
	(OR)			
	12.b.	Let $f: X \rightarrow Y$. If the function f is continuous then examine that for every convergent sequence $x_n \rightarrow x$ in X the sequence $f(x_n)$ converges to $f(x)$. The converse holds if X is metrizable?		
3	13.a.	Prove that the image of a connected space under a continuous map is connected?	K3	CO3
	(OR)			
	13.b.	Let X be a topological space. Then examine whether X is compact if and only if for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property the intersection $\bigcap_{C \in \mathcal{C}} C$ of all the elements of $\mathcal{C}$ is nonempty?		
4	14.a.	Suppose that X has a countable basis then verify the following (i) Every open covering of X contains a countable subcollection covering X (ii) There exists a countable subset of X that is dense in X ?	K4	CO4
	(OR)			
	14.b.	Verify that a subspace of a Hausdroff space is Hausdroff and product of Hausdroff space is Hausdroff?		
5	15.a.	Prove that every compact Hausdroff space is normal?	K5	CO5
	(OR)			
	15.b.	State and prove Tychonoff theorem?		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let X be a topological space then show that the following conditions hold (i) $\emptyset$ and X are closed (ii) Arbitrary intersection of closed sets are closed (iii) Finite unions of closed sets are closed	K4	CO1
2	17	Show that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are same as the product topology on $\mathbb{R}^n$?	K5	CO2
3	18	Prove that a finite Cartesian product of connected space is connected?	K5	CO3
4	19	If X be a metrizable space then verify the following (i) X is compact (ii) X is limit point compact (iii) X is sequentially compact	K4	CO4
5	20	Prove that every well ordered set X is normal in the order topology?	K4	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - MATHEMATICS

DIFFERENTIAL GEOMETRY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The binomial line at P is the normal in a direction orthogonal to the _____. a) torsion b) osculating plane c) curvature d) evolute	K1	CO1
	2	The involutes of a circular helix are _____. a) curvature b) circle c) sphere d) plane curves	K2	CO1
2	3	The parametric longitude curves $v=\text{constant}$. a) Meridians b) Parallels c) Revolution d) Helicoid	K1	CO1
	4	One period of a _____ of pitch $2\pi a$ corresponds isometrically to a whole catenoid of parameter. a) Helicoid b) Right helicoid c) Curvature d) Torson	K2	CO1
3	5	Every helix on a cylinder is a _____. a) Curvature b) Geodesic c) Catenoid d) Meridians	K1	CO1
	6	Two surfaces of the same constant curvature are _____. a) Isometric b) Locally isometric c) Orthogonal d) Parallel	K2	CO1
4	7	If ϕ denotes the angle between the principal normal $\vec{n}$ to a curve on the surface and the surface normal $\vec{N}$ then $k\cos\phi = K_n$. This is known as a) Rodrigue's theorem b) Euler theorem c) Meusnier theorem d) Dupin indication theorem	K1	CO1
	8	_____ contains all three types of points. a) Catenoid b) Helicoids c) Bertrand curves d) Anchor ring	K2	CO1
5	9	The only compact surface of class ≥ 2 for which every point is an umbilic are _____. a) Spheres b) Helices c) Anchor ring d) Circles	K1	CO1
	10	In a closed region R of a surface of constant positive _____ curvature without umbilicus, the principal curvatures take their extreme values at the boundary. a) Gaussian b) Principal c) Mean d) Geodesic	K2	CO1

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the length of the curve given as the intersection of the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a \cosh(z/a)$ from the point $(a, 0, 0)$ to the point (x, y, z) .	K3	CO2
		(OR)		
	11.b.	State and prove Serret-Frenet Formulae.		
2	12.a.	Derive the angle between parametric curves.	K3	CO2
		(OR)		
	12.b.	On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.		
3	13.a.	A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.	K3	CO2
		(OR)		
	13.b.	Prove that if the orthogonal trajectories of the curves $v = \text{constant}$ are geodesics, and then $\frac{H^2}{E}$ is independent of u .		
4	14.a.	State and prove Meusnier's theorem.	K3	CO3
		(OR)		
	14.b.	Prove that the edge of regression of the rectifying developable has equation $\bar{R} = \bar{r} + \kappa \frac{(\bar{\pi} + \kappa \bar{b})}{(\kappa' \tau - \kappa \tau')}$		
5	15.a.	Prove that the only compact surfaces whose Gaussian curvature is positive and mean curvature constant are spheres.	K3	CO3
		(OR)		
	15.b.	Explain complete surfaces.		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that the radius of the curvature of the locus of the Centre of curvature of a curve is given by $\left[\left\{ \frac{\rho^2 \sigma}{R^3} \frac{d}{ds} \left(\frac{\sigma \rho'}{\rho} \right) - \frac{1}{R} \right\}^2 + \frac{\rho'^2 \sigma^4}{\rho^2 R^4} \right]^{-1/2}$	K4	CO4
2	17	Find a surface of revolution which is isometric with a region of the right helicoid?	K4	CO4
3	18	State and prove Gauss Bonnet theorem.	K4	CO4
4	19	Derive Rodrigues' formula.	K4	CO5
5	20	Prove that the only compact surface of class ≥ 2 for which every point is an umbilic are spheres.	K4	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch – MATHEMATICS

FUNCTIONAL ANALYSIS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The null space of T is the set of all $x \in D(T)$ such that _____. a) $Tx \neq 0$ b) $Tx = 0$ c) $T(\alpha x) = \alpha T(x)$ d) $T0 = 0$	K1	CO1
	2	If a normed space has a Schauder basis then it is _____. a) separable b) absolutely convergent c) schauder basis d) convex set	K2	CO1
2	3	If $x \perp y$ in an inner product space X then $\ x + y\ ^2 =$ _____. a) $\ x\ ^2$ b) $\ y\ ^2$ c) $\ x - y\ ^2$ d) $\ xy\ ^2$	K1	CO2
	4	If $x(t) = \begin{cases} t & \text{if } -\pi/2 \leq t < \pi/2 \\ \pi - t & \text{if } \pi/2 \leq t < 3\pi/2 \end{cases}$, then $a_K =$ _____. a) 0 b) 1 c) 2π d) $4/\pi$	K2	CO2
3	5	A chain is a partially ordered set such that every two elements of the set are _____. a) incomparable b) comparable c) partially ordered d) totally ordered	K1	CO3
	6	A maximal element of M is an $m \in M$ such that $m \leq x$ implies _____. a) mx b) $m \neq x$ c) $m = x$ d) $m = 0$	K2	CO3
4	7	If X is _____ with a subspace Z then X is said to be embeddable in a normed space. a) isomorphic b) homeomorphic c) homomorphic d) unique fixed point	K1	CO4
	8	A vector space X is algebraically reflexive if the canonical mapping $C: X \rightarrow X^{**}$ is _____. a) injective b) bijective c) surjective d) reflexive	K2	CO4
5	9	A mapping $T: X \rightarrow X$ is a contraction on X if there is a positive real number $\alpha < 1$ such that for all $x, y \in X$ then _____. a) $d(Tx, Ty) < \alpha d(x, y)$ b) $d(Tx, Ty) \leq \alpha d(x, y)$ c) $d(Tx, Ty) = d(x, y)$ d) $d(Tx, Ty) \geq \alpha d(x, y)$	K1	CO4
	10	If there is a constant K such that for all $x, y \in [a, b]$, $ Tx - Ty \leq K x - y $ then the mapping $T: [a, b] \rightarrow [a, b]$ is said to satisfy a _____. a) Fixed point b) Contraction c) Libschitz condition d) Decreasing map	K2	CO4

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that a subspace Y of a Banach Space X is complete if and only if the set Y is closed in X .	K2	CO1
		(OR)		
	11.b.	If a normed space X is finite dimensional then Prove that every linear operator on X is bounded.		
2	12.a.	State and Prove Schwarz inequality.	K3	CO2
		(OR)		
	12.b.	Let Y be any closed subspace of a Hilbert space H then $H = Y \oplus Z$, $Z = Y^\perp$.		
3	13.a.	State and Prove the Riesz representation theorem.	K3	CO3
		(OR)		
	13.b.	Prove that an every Hilbert Space $H \neq \{0\}$ there exists a total orthonormal set.		
4	14.a.	If the dual space X' of a normed space X is separable then prove that X itself is separable.	K4	CO4
		(OR)		
	14.b.	State and Prove Uniform boundedness theorem.		
5	15.a.	Prove that the range $\mathcal{R}(T)$ of a compact linear operator $T: X \rightarrow Y$ is separable if X & Y are normed spaces.	K4	CO4
		(OR)		
	15.b.	Let (T_n) be a sequence of compact linear operator from a normed space X into a Banach space Y . If (T_n) is uniformly operator convergent, say $\ T_n - T\ \rightarrow 0$ then the limit operator T is compact.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that an every finite dimensional subspace Y of a normed space X is complete. In particular every finite dimensional normed space is complete.	K4	CO1
2	17	State and Prove Bessel's Inequality.	K4	CO2
3	18	State & prove Hahn Banach Theorem.	K4	CO3
4	19	Prove that a bounded linear operator T from a Banach space X on a Banach space Y is an open mapping. Hence if T is bijective, T^{-1} is continuous and then bounded.	K4	CO4
5	20	Let $T: X \rightarrow Y$ be a compact linear operator on a normed space X then for every $\lambda \neq 0$, the range of $T_\lambda = T - \lambda I$ is closed.	K4	CO4

Z-Z-Z

END

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)**

Branch - **MATHEMATICS**

MECHANICS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks

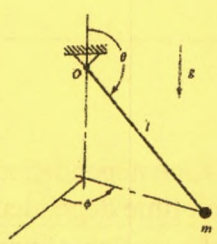
(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Scleronomous constraints have ____ a) explicit time dependence b) no explicit time dependence c) both explicit time dependence and no explicit time dependence d) neither explicit time dependence nor no explicit time dependence.	K1	CO1
	2	The number of coordinates required to describe the motion of particle at any time are called ____ a) phase coordinates b) space coordinates c) generalized coordinates d) polar coordinates	K2	CO1
2	3	The Lagrange's equation of motion is given as ____ a) $\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial q_j} \right) - \frac{\partial L}{\partial q_j} = 0$ b) $\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial q_j} \right) + \frac{\partial L}{\partial q_j} = 0$ c) $\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial q_j} \right) + \frac{\partial L}{\partial q_j} = 0$ d) $\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial q_j} \right) - \frac{\partial L}{\partial q_j} = 0$	K1	CO2
	4	____ has succeeded in eliminating the ignorable co-ordinates from the equation of motion. a) Lagrangian equation b) Legendre procedure c) Routhian procedure d) Hamilton equation	K2	CO2
3	5	The variational principle of most importance in dynamics is ____ a) Lagrangian principle b) stationary c) Hamilton's principle d) potential energy	K1	CO2
	6	To apply the ____ method to problem involving constrained minima or maxima. a) Lagrangian multiplier b) Hamilton's c) Legendre d) variational	K2	CO2
4	7	The complete solution of Hamilton's canonical equations commonly known as the solution of the ____ a) Hamilton's problem b) Lagrangian problem c) Hamilton-Jacobi equation d) generating function	K1	CO3
	8	The generating function is the solution of the partial differential equation known as the ____ a) Hamilton Principle b) Canonical Integral c) Pfaffian Differential form d) Hamilton-Jacobi equation	K2	CO3
5	9	____ systems which can be described by the standard form of either Hamilton's or Lagrange's equations. a) stationary b) non holonomic c) holonomic d) point transformation	K1	CO4
	10	Homogenous canonical transformation is also known as a ____ transformation. a) Mathieu b) Hamilton-Jacobi c) exact differential d) Lagrange's	K2	CO4

Cont...

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	What are the generalized coordinates? Give an example of a transformation from Cartesian to generalized coordinates. Explain it.	K2	CO1
	(OR)			
	11.b.	Discuss the nature of a virtual displacement.		
2	12.a.	Find the differential equations of motion for a spherical pendulum of length l in figure. 	K3	CO2
	(OR)			
	12.b.	Discuss the Liouville's system to find the constants of the motion.		
3	13.a.	Find the path of minimum length between two given points on the two-dimensional surface of a sphere of radius r .	K3	CO2
	(OR)			
	13.b.	Find the equations of motion for a charged particle in an electromagnetic field.		
4	14.a.	What is the form of the Pfaffian differential equations? Explain it.	K4	CO3
	(OR)			
	14.b.	Show that the Liouville conditions are sufficient for the separability of an orthogonal system.		
5	15.a.	The transformation $Q = \frac{1}{2}(q^2 + p^2)$, $P = -\tan^{-1} \frac{q}{p}$ Show that this transformation is canonical.	K4	CO4
	(OR)			
	15.b.	The transformation $Q = \sqrt{e^{-2q} - p^2}$, $P = \cos^{-1}(Pe^q)$ Use the Poisson bracket to show that it is canonical.		

SECTION - C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	A particle of mass m is suspended by a massless wire of length $r = a + b \cos \omega t$ ($a > b > 0$) to form a spherical pendulum. Find the equations of motion.	K4	CO1
2	17	Derive Lagrange's equations of motion.	K4	CO2
3	18	Find the necessary conditions for a stationary value of $I = \int_{x_0}^{x_1} f[y(x), y'(x), x] dx$	K4	CO2
4	19	Explain the canonical integral associated with Hamilton's principal.	K4	CO3
5	20	Obtain the four major types of generating functions associated with this transformation $Q = \log \frac{\sin p}{q}$ $P = q \cot p$	K4	CO4

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024

(Third Semester)

Branch - MATHEMATICS

MAJOR ELECTIVE COURSE – II STOCHASTIC DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	An optimal stopping problem can also be expressed in terms of a set of ____ a) variational inequalities b) distributions c) expected profit d) none	K1	CO1
	2	The Mathematical model for a random quantity is a ____ a) σ - algebra b) measurable set c) random variable d) Probability measure	K2	CO1
2	3	Itô integral is defined by ____ a) $I[f](w) = \int_S^T f(t, w) dB_t(w)$ b) $I[f](w) = \int_S^T f(t, w) dB(w)$ c) $I[f] = \int_S^T f(t, w) dB_t(w)$ d) $I[f](w) = \int_T^S f(t, w) dB_t(w)$	K1	CO2
	4	$N_t = B_t^3 - 3t B_t$ is a ____ a) Brownian motion b) Martingale c) iterated Itô integrals d) none	K2	CO2
3	5	The solution X_t found above is called a ____ solution, because the variation B_t of Brownian motion is given in advance and the solution X_t constructed from it is Ft^z - adapted. a) Weak b) General c) Strong d) Classical	K1	CO3
	6	Itô processes is also called the ____ a) Brownian motion b) Stochastic process c) Stochastic integrals d) none	K2	CO3
4	7	An expression for X_t by solving the (linear) stochastic differential equation ____ a) $dX_t = F(t)X_t dt + C(t)dU_t$ b) $dX_t = F(t)X_t dt + C(t)dt$ c) $dX_t = F(t)dt + C(t)dU_t$ d) $dX_t = F(t)X_t dt + C(t)dU$	K1	CO4
	8	For the solution of linear filtering problems governed by more general processes then ____ a) dimension on the motion b) invertible c) Brownian motion d) asymptotic estimation	K2	CO4
5	9	The harmonic measure of X on $\partial G, \mu_G^x$, is ____ a) $\mu_G^x(F) = Q^x[X_{\tau G} \in F]$ b) $\mu_G^x(F) = [X_{\tau G} \in F]$ c) $\mu_G^x(F) = Q^x[X_G \in F]$ d) $\mu_G^x(Q) = F[X_{\tau G} \in F]$	K1	CO5
	10	If $Z(t)$ is a lower bounded local martingale, then $Z(t)$ is a ____ a) local martingale b) uniformly integrals c) super martingale d) N_t -martingale	K2	CO5

Cont...

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove Kolmogorov's continuity theorem.	K3	CO1
		(OR)		
	11.b.	If $X, Y: \Omega \rightarrow R^n$ are two given functions, then prove that Y is H_X -measurable if and only if there exists a Borel measurable function $g: R^n \rightarrow R^n$ such that $Y = g(X)$.		
2	12.a.	Let $f \in v(0, T)$. Then prove that there exists a t -continuous version of $\int_0^t f(s, w) dB_s(w)$; $0 \leq t \leq T$, i.e. there exists a t -continuous stochastic process J_t on $(\Omega, \mathcal{F}, P)$ such that $P \left[J_t = \int_0^t f dB \right] = 1$ for all $t, 0 \leq t \leq T$.	K2	CO2
		(OR)		
	12.b.	Write a comparison of Itô and Stratonovich integrals.		
3	13.a.	State and prove the Itô representation theorem.	K3	CO3
		(OR)		
	13.b.	State and prove existence and uniqueness theorem for stochastic differential equations.		
4	14.a.	Prove that $M_t = \begin{bmatrix} X_t \\ Z_t \end{bmatrix} \in R^2$ is a Gaussian process.	K4	CO4
		(OR)		
	14.b.	Let $X, Z_s; s \leq t$ be random variables in $L^2(P)$ and assume that $X, Z_{s_1}, Z_{s_2}, \dots, Z_{s_n} \in R^{n+1}$ has a normal distribution for all $s_1, s_2, \dots, s_n \leq t, n \geq 1$. Then prove that $\mathcal{P}_L(X) = E[X G] = \mathcal{P}_K(X)$.		
5	15.a.	If x is not a trap for X_t , then prove that there exists an open set $U \ni x$ such that $E^x[J_U] < \infty$.	K4	CO5
		(OR)		
	15.b.	State and prove the theorem on strong Markov property for Itô diffusions.		

SECTION -C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Explain Brownian motion with an example.	K4	CO1
2	17	State and prove the Itô isometry lemma.	K4	CO2
3	18	State and prove the theorem on Integration by parts.	K4	CO3
4	19	Prove that i) N_t has orthogonal increments ii) $E[N_t^2] = \int_0^t D^2(s) ds$ iii) $\mathcal{L}(N, t) = \mathcal{L}(Z, t)$ for all $t \geq 0$ iv) N_t is a Gaussian process	K4	CO4
5	20	State and prove the theorem on Markov property for Itô diffusions.	K4	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - MATHEMATICS

MAJOR ELECTIVE COURSE – II NUMBER THEORY AND CRYPTOGRAPHY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	The symbol $\binom{n}{k}$ is given by _____. (a) $\frac{k!}{n!(n-k)!}$ (b) $\frac{n!}{k!(n-k)!}$ (c) $\frac{n}{k(n-k)}$ (d) $\frac{n!}{k!(k-n)!}$	K1	CO1
2	$(87,27)=$ _____. (a) 1 (b) 2 (c) 3 (d) 4	K2	CO1
3	A natural number, p is said to be _____ number if $p>1$ and p has no non-trivial divisor. (a) composite (b) real (c) rational (d) prime	K1	CO2
4	If $(a,b)=1$, then there are infinitely many primes of the form _____. (a) $a+bn$ (b) $an+b$ (c) $a+b$ (d) $a+4b$	K2	CO2
5	If $a \equiv b \pmod{m}$, then _____. (a) $ac \equiv bc \pmod{m}$ (b) $ac \equiv b \pmod{m}$ (c) $a \equiv bc \pmod{m}$ (d) $a \equiv b \pmod{cm}$	K1	CO3
6	If a and b belong to Z_m , then _____ is the least positive residue $(\pmod{m})$ of ab. (a) $a \oplus b$ (b) $a \oslash b$ (c) $a \cdot b$ (d) $a * b$	K2	CO3
7	$\sigma(n)$ is _____. (a) associative (b) commutative (c) multiplicative (d) zero	K1	CO4
8	n is _____ if $\sigma(n) = 2n$. (a) perfect (b) square (c) multiplicative (d) composite	K2	CO4
9	The basic function of cryptography is _____. (a) data compression (b) data encryption (c) data storage (d) data retrieval	K2	CO5
10	Which algorithm is widely known for public key encryption? (a) DES (b) AES (c) RSA (d) BSA	K2	CO5

Cont...

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Prove that $V_n^2 - Du_n^2 = 4Q^n$.	K3	CO1
	(OR)		
11.b.	State and prove principle of mathematical induction.		
12.a.	If p is prime and $p ab$, then prove $p a$ or $p b$.	K3	CO2
	(OR)		
12.b.	If n is composite, and if p is atleast prime factor of n, then prove that $p \leq \sqrt{n}$.		
13.a.	If $ac \equiv bc \pmod{m}$ and $(c,m)=1$, then prove that $a \equiv b \pmod{m}$.	K3	CO3
	(OR)		
13.b.	State and prove Wilson's theorem.		
14.a.	If f is an arithmetic function such that $f(1) \neq 0$, then prove that f^{-1} exists.	K3	CO4
	(OR)		
14.b.	State and prove Euler's Criterion.		
15.a.	Explain digraph transformations.	K2	CO5
	(OR)		
15.b.	Find the inverse of $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in M_2(\mathbb{Z}/26\mathbb{Z})$.		

SECTION - C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Question No.	Question	K Level	CO
16	State and prove division algorithm.	K3	CO1
17	If $n > 1$, then prove that the canonical factorization of n is unique.	K3	CO2
18	State and prove Chinese remainder theorem.	K3	CO3
19	State and prove Gauss' lemma.	K3	CO4
20	Discuss the differences between classical and public key in cryptography.	K2	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – MATHEMATICS

MATHEMATICAL METHODS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. If $D(\lambda) \neq 0$, then an integral equation has ----- solutions.
a) only one b) two c) many d) zero
2. The inhomogeneous Fredholm Integral equation with separable kernel has ----- solution.
a) zero b) one
c) two d) infinite
3. An initial value problems in ordinary differential equations leads to a
a) Volterra I.E of second kind b) Fredholm I.E of second kind
c) Volterra I.E of first kind d) Fredholm I.E of first kind
4. If F is independent of y' , then the Euler equation has the form-----
a) $F_y(x,y) = 0$ b) $F_x(x,y) = 0$ c) $F_{y'} = 0$ d) $F_{y'y'} = 0$
5. If the pencil of curves intersect at a common point, then it forms a
(a) field (b) central field
(c) proper field (d) none of these

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6a) Show that the integral equation $g(s) = f(s) + \frac{1}{\pi} \int_0^{2\pi} (\sin(s+t))g(t)dt$ possesses no solution for $f(s) = s$, but that it possesses infinitely many solutions when $f(s) = 1$.
OR
- b) Find the resolvent kernel for the Integral equation $g(s) = f(s) + \int_{-1}^1 (st + s^2 t^2)g(t)dt$
- 7 a) Find the Neumann series for the solution of the integral equation.
 $g(s) = (1+s) + \lambda \int_0^1 (s-t)g(t)dt$.
OR
- b) Find the eigen values and eigen functions of the homogeneous integral equation
 $g(s) = \lambda \int_0^1 (st + \frac{1}{st})f(t)dt$.
- 8 a) Convert an Initial value problem to the volterra integral equation of II kind.
OR
- b) Reduce the initial value problem $y''(s) + \lambda y(s) = F(s)$, $y(0)=1$, $y'(0)=0$ to volterra integral equation.

Cont....

- 9 a) On what curves can the functional $v(y(x)) = \int_0^1 (y'^2 + 12xy) dx$ $y(0)=0, y(1)=1$ be extremized?

OR

- b) State and prove the fundamental lemma of the calculus of variations.

- 10 a) Test for an extremum the functional $v(y(x)) = \int_0^a y'^3 dx$, $y(0)=0, y(a)=b, a>0, b>0$.

OR

- b) Test for an extremum of the functional $v(y(x)) = \int_0^{x_1} \left(\frac{\sqrt{1+y'^2}}{\sqrt{y}} \right) dx$, $y(0)=0, y(x_1)=y_1$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11a) Solve the Fredholm Integral equation of the second kind $g(s) = \lambda + \int_0^1 (s+t)g(t)dt$

OR

- b) Solve the integral equations using Approximate method

$$G(s) = e^s - s - \int_0^1 s(e^{st} - 1)g(t)dt.$$

- 12 a) State and prove Fredholm's First theorem on Integral equation.

OR

- b) Solve the integral equation $g(s) = s + \lambda \int_0^1 (st + \sqrt{st})g(t)dt$ using Fredholm's first theorem.

- 13 a) Convert a boundary value problem to the Fredholm integral equation of II kind

OR

- b) Solve the integral equation $f(s) = \int_0^1 \frac{g(t)}{(\cos t - \cos s)^{1/2}} dt$.

- 14 a) Derive Euler Equation of the functional $F(x, y, y')$.

OR

- b) Prove that the arc length of the curve $I[y(x)] = \int_{x_0}^{x_1} \sqrt{1+y'^2} dx$ has extremals are straight lines $y=c_1 x+c_2$.

- 15 a) Test for an extremum the functional $v(y(x)) = \int_0^a \frac{y}{y'^2} dx, y(0)=1, y(a)=b, a>0, 0<b<1$.

OR

- b) Is Jacobi condition fulfilled for the extremal of the functional

$$v(y(x)) = \int_0^{\pi/2} (y'^2 + y^2 + x^2) dx \text{ that passes through the point } A(0,0) \text{ and } B(a,0).$$

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – MATHEMATICS

ALGEBRA

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$a \in Z$ if and only if $N(a) = \underline{\hspace{1cm}}$. a) $\emptyset$ b) $\{e\}$ c) G d) $G - \{e\}$	K1	CO1
	2	If $p^m/o(G), p^{m+1} \nmid o(G)$, then G has a subgroup of order _____. a) p b) p^m c) p^{m+1} d) none of these	K2	CO1
2	3	Let F be a field and $f(x) \in F[x]$ be a polynomial. Then the number of distinct roots of $f(x)$ in F is _____ $\deg(f)$. a) at least b) equal to c) at most d) greater than	K1	CO2
	4	If F is a field then $F[x_1, x_2, \dots, x_n]$ is a _____. a) field b) integral domain c) unique factorization domain d) Euclidean domain	K2	CO2
3	5	If $a \in K$ is algebraic over F , then $[F(a):F] = \underline{\hspace{1cm}}$. a) n b) $n - 1$ c) $n + 1$ d) n^2	K1	CO3
	6	If a and b are algebraic over F of degrees m and n respectively, then $a - b$ is also algebraic of degree at most _____. a) $m + n$ b) $m - n$ c) mn d) $n - m$.	K2	CO3
4	7	Any field is perfect if its characteristic is _____. a) 0 b) 1 c) prime d) non-zero	K1	CO4
	8	If $K = C$ and $F = R$, then the fixed field of $G(K, F)$ is _____. a) K b) F c) C d) K/F	K2	CO4
5	9	The characteristic roots of $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ is _____. a) 2, 3 b) 2, -3 c) -2, 3 d) -2, -3	K1	CO5
	10	If $T \in A(V)$ such that $T^* = T$ then T is called _____. a) unitary b) Hermitian c) symmetric d) normal	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	If $o(G) = p^n$, where p is a prime number, then prove that $Z(G) \neq \{e\}$.	K3	CO1
		(OR)		
	11.b.	Develop the proof that for every integer s , $G(s)$ and $G'(s)$ are isomorphic if G and G' are isomorphic abelian groups.		

Cont...

2	12.a.	Inspect the statement and proof of division algorithm for polynomials.	K4	CO2
	(OR)			
	12.b.	Test for the nature of product of two primitive polynomials.		
3	13.a.	If L is an algebraic extension of K and K is an algebraic extension of F , then prove that L is an algebraic extension of F .	K5	CO3
	(OR)			
	13.b.	State and prove the remainder theorem.		
4	14.a.	The polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.-Examine.	K4	CO4
	(OR)			
	14.b.	If K is a finite extension of F , then analyze whether $G(K, F)$ is a finite group and its order $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K: F]$.		
5	15.a.	If V is n -dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, \dots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, \dots, w_n$ of V over F , then identify an element $C \in F_n$ such that $m_2(T) = C m_1(T) C^{-1}$.	K3	CO5
	(OR)			
	15.b.	Apply the necessary concepts to prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V .		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

 $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Analyze the statement and proof of the third part of Sylow's theorem.	K4	CO1
2	17	Explain Eisenstein Criterion theorem.	K5	CO2
3	18	Examine that a polynomial of degree n over a field can have at most n roots in any extension field.	K4	CO3
4	19	Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F .	K4	CO4
5	20	If V is n -dimensional over F and if $T \in A(V)$ has all its characteristic roots in F , then justify that T satisfies a polynomial of degree n over F .	K5	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch - MATHEMATICS

REAL ANALYSIS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	A real function f has a local maximum at a point $p \in X$ if there exists $\delta > 0$ such that _____ for all $q \in X$ with $d(p, q) < \delta$. (a). $f(p) \leq f(q)$ (b). $f(q) \leq f(p)$ (c). $f(p) < f(q)$ (d). $f(q) < f(p)$	K1	CO1
	2	Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that _____. (a). $f'(x) > \lambda$ (b). $f'(x) < \lambda$ (c). $f'(x) \neq \lambda$ (d). $f'(x) = \lambda$	K2	CO1
2	3	Given two partitions P_1 and P_2 , then P^* is their common refinement if _____. (a). $P^* = P_1 \cup P_2$ (b). $P^* = P_1 \cap P_2$ (c). $P^* < P_1 \cup P_2$ (d). $P^* > P_1 \cup P_2$	K1	CO2
	4	If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and if $ f(x) \leq M$ on $[a, b]$, then $\left \int_a^b f d\alpha \right $ is _____. (a). $\geq M[\alpha(b) - \alpha(a)]$ (b). $> M[\alpha(b) - \alpha(a)]$ (c). $\leq M[\alpha(b) - \alpha(a)]$ (d). $< M[\alpha(b) - \alpha(a)]$	K2	CO2
3	5	The sequence $\{f_n\}$ is uniformly bounded on E if there exists a number M such that _____, $x \in E, n = 1, 2, 3, \dots$ (a). $ f_n(x) \geq M$ (b). $ f_n(x) > M$ (c). $ f_n(x) \leq M$ (d). $ f_n(x) < M$	K1	CO3
	6	Every _____ sequence is pointwise convergent. (a). uniformly convergent (b). uniformly bounded (c). pointwise bounded (d). none of these	K2	CO3
4	7	Let e^x be defined on $\mathbb{R}^1$. Then $\lim_{x \rightarrow \infty} x^n e^{-x} = ______$, for every n . (a). ∞ (b). $-\infty$ (c). 0 (d). none	K1	CO4
	8	$\beta(x, y) = ______$ (a). $\frac{\Gamma(x)\Gamma(y)}{\Gamma(x-y)}$ (b). $\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ (c). $\frac{\Gamma(x+y)}{\Gamma(x)\Gamma(y)}$ (d). $\frac{\Gamma(x-y)}{\Gamma(x)\Gamma(y)}$	K2	CO4
5	9	Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then $\dim X$ _____. (a). $\leq r$ (b). $< r$ (c). $\geq r$ (d). $> r$	K1	CO5
	10	Let X be a metric space, with distance d . If ϕ maps X into X and if there is a number $c < 1$ such that $d(\phi(x), \phi(y))$ is _____ for all $x, y \in X$, then ϕ is said to be a contraction of X into X . (a). $< c d(x, y)$ (b). $\geq c d(x, y)$ (c). $> c d(x, y)$ (d). $\leq c d(x, y)$	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Suppose f and g are defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$. Then prove that $f + g$, fg and f/g are differentiable at x .	K2	CO1
		(OR)		
	11.b.	State and prove the mean value theorem.		
2	12.a.	Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.	K2	CO2
		(OR)		
	12.b.	If f maps $[a, b]$ into $\mathbb{R}^k$ and if $f \in \mathcal{R}(\alpha)$ for some monotonically increasing function α on $[a, b]$, then prove that $ f \in \mathcal{R}(\alpha)$, and $ \int_a^b f d\alpha \leq \int_a^b f d\alpha$.		
3	13.a.	State and prove the Cauchy criterion for uniform convergence.	K1	CO3
		(OR)		
	13.b.	Prove that there exists a real continuous function on the real line which is nowhere differentiable.		
4	14.a.	Suppose $\sum c_n$ converges. $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $-1 < x < 1$. Then show that $\lim_{x \rightarrow 1} f(x) = \sum_{n=0}^{\infty} c_n$.	K3	CO4
		(OR)		
	14.b.	Suppose $a_0, \dots, a_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$, $P(z) = \sum_{k=0}^n a_k z^k$. Then prove that $P(z) = 0$ for some complex number z .		
5	15.a.	Let Ω be the set of all invertible linear operations on $\mathbb{R}^n$, prove that the following: (i) If $A \in \Omega$, $B \in L(\mathbb{R}^n)$, and $\ B - A\ \cdot \ A^{-1}\ < 1$, then $B \in \Omega$. (ii) Ω is an open subset of $L(\mathbb{R}^n)$, and the mapping $A \rightarrow A^{-1}$ is continuous on Ω .	K3	CO5
		(OR)		
	15.b.	If X is a complete metric space and if φ is a contraction of X into X , then prove that there exists one and only one $x \in X$ such that $\varphi(x) = x$.		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Taylor's theorem.	K2	CO1
2	17	Assume α increases monotonically and $\alpha \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.	K3	CO2
3	18	State and prove the Stone-Weierstrass theorem.	K1	CO3
4	19	State and prove the Parseval's theorem.	K4	CO4
5	20	State and the implicit function theorem.	K4	CO5

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – MATHEMATICS

MATHEMATICAL STATISTICS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module	Question No.	Question	K Level	CO								
1	1	The table of probability distribution of a random variable X is given below. <table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(x_i)$</td><td>$\frac{1}{3}$</td><td>K</td><td>$\frac{1}{6}$</td></tr></table> Then the expectation of X is _____. (a) $\frac{5}{6}$ (b) 1 (c) 0 (d) $1/6$	x_i	0	1	2	$P(x_i)$	$\frac{1}{3}$	K	$\frac{1}{6}$	K1	CO1
	x_i	0	1	2								
$P(x_i)$	$\frac{1}{3}$	K	$\frac{1}{6}$									
	2	The value of $\phi(0) =$ _____. (a) 1 (b) -1 (c) 0 (d) ∞	K2	CO1								
2	3	In a binomial distribution the mean is 4 and the variance is $\frac{8}{3}$. Then the values of n and p are respectively _____. (a) $12, \frac{1}{3}$ (b) $12, \frac{2}{3}$ (c) $16, \frac{1}{3}$ (d) $12, \frac{1}{9}$	K1	CO2								
	4	In a poisson distribution, if $P(X=0)=k$ then the variance is _____. (a) e^k (b) e^{-k} (c) $\log_e k$ (d) $\log_e \left(\frac{1}{k}\right)$	K2	CO2								
3	5	The sequence of random variables $\{Z_n\} = \{X_n - X\}$ is stochastically convergent to _____. (a) 1 (b) $-\infty$ (c) ∞ (d) 0	K1	CO3								
	6	The random variable X_n has an asymptotically normal distribution with standard deviation is _____. (a) npq (b) np (c) $\sqrt{npq}$ (d) $\sqrt{np}$	K2	CO3								
4	7	If the process is real and $m = 0, \sigma^2 = 1$, then $R(\tau)$ is called _____ function. (a) Correlation (b) Regression (c) Moment (d) Stationary	K1	CO4								
	8	If j is transient, then as $n \rightarrow \infty, p_{jj}^{(n)} \rightarrow$ _____. (a) 1 (b) 0 (c) ∞ (d) -1	K2	CO4								
5	9	Find the number of degrees of freedom in a 3×3 contingency table is : (a) 8 (b) 4 (c) 3 (d) 5	K1	CO5								
	10	Analysis of variance (ANOVA) performs which type of test? (a) F-test (b) t-test (c) z-test (d) xy-test	K2	CO5								

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11a.	State and prove Chebyshev inequality.	K5	CO1
		(OR)		
	11.b.	The random variable X can take on the values $x_1 = -1$ and $x_2 = 1$ with probabilities $P(X = -1) = P(X = 1) = 0.5$. Determine the characteristic function.		

Cont...

Cont...

2	12.a.	The random variable X has the distribution $N(1; 2)$. Find the probability that X is greater than 3.	K1	CO2
	(OR)			
	12.b.	Obtain the moments of the normal distribution.		
3	13.a.	We throw a coin $n = 100$ times. We assign the number 1 to the appearance of heads & the number 0 to the appearance of tails. The probability of each of these events is equal to $p = q = 0.5$. what is the probability that heads will appear more than 50 times & less than 60 times ?	K1	CO3
	(OR)			
	13.b.	Let $F_n(x)$ ($n = 1, 2, \dots$) be the distribution function of the random variable X_n . Prove that the sequence $\{X_n\}$ is stochastically convergent to zero iff the sequence $\{F_n(x)\}$ satisfies the relation $\lim_{n \rightarrow \infty} F_n(x) = \begin{cases} 0 & \text{for } x < 0, \\ 1 & \text{for } x > 0. \end{cases}$		
4	14.a.	Examine that, A process stationary in the wide sense is continuous iff its covariance function $R(\tau)$ is continuous at zero.	K4	CO4
	(OR)			
	14.b.	Analyze the statement "Let $\{X_t, 0 \leq t < \infty\}$ be a homogeneous markov process with a finite number of states $i = 0, 1, 2, \dots, s$. If there exists a $t^* (0 \leq t^* < \infty)$ such that $P_{ij}(t^*) > 0$ ($i, j = 0, 1, \dots, s$), then the limits $\lim_{t \rightarrow \infty} p_{ij}(t) = p_j > 0$ ($i, j = 0, 1, \dots, s$) exist".		
5	15.a.	Prove that - The sequence $\{F_n(t)\}$ of distribution functions of student's t with n degrees of freedom satisfies for every t the relation $\lim_{n \rightarrow \infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-t^2/2} dt.$	K5	CO5
	(OR)			
	15.b.	From a population in which the characteristic X has the normal distribution $N(1; 2)$, we draw a simple sample of size $n = 12$. We observe the following values of X : $x_1 = 2.0, x_2 = 2.5, x_3 = 0.5, x_4 = 1.0, x_5 = 0.0, x_6 = -0.9, x_7 = 5.1, x_8 = -1.5, x_9 = 0.8, x_{10} = 1.1, x_{11} = 0.8, x_{12} = 0.4$. Estimate the probability that Z will exceed or equal the value obtained, = 32.28 ?		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	The characteristic function of the random variable X is given by the formula $\phi(t) = \exp(-\frac{t^2}{2})$. Find the density function of this random variable.	K1	CO1
2	17	Denote by V the velocity of a particle of an ideal gas. Consider the rectangular system of co-ordinates x_1, x_2, x_3 . Let $\vec{V}$ be the velocity vector of the particle, & V_1, V_2, V_3 its projections on the corresponding axes. The position & length of the vector $\vec{V}$ have a random character; hence (V_1, V_2, V_3) is a three-dimensional random variable. Determine its density function.	K5	CO2
3	18	State and prove De moivre-Laplace theorem.	K3	CO3
4	19	Analyze the statement "A function $R(\tau)$ is the correlation function of a process $\{Z_t, -\infty < t < +\infty\}$ stationary in the wide sense, continuous, & satisfying $m = 0, \sigma = 1$ iff there exists a distribution function $F(\lambda)$ such that $R(\tau) = \int_{-\infty}^{+\infty} e^{i\lambda\tau} dF(\lambda)$ ".	K4	CO4
5	20	Derive the Snedecor's F test from Fisher's Z test.	K5	CO5

Z-Z-Z END