

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMEBR 2024
(First Semester)

Branch - MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours

Maximum: 75 Marks

Q. No.	SECTION A (10 x 1 = 10) Answer ALL Questions	Level	CO
1.	The equation $x'' - 2tx' + 2nx = 0$ is known as (a) Bessel equation (b) Hermite equation (c) Legendre equation (d) None of the above	K2	CO1
2.	The Bessel equation possesses a regular singular point at — (a) 0 (b) 1 (c) -1 (d) 2	K1	CO1
3.	A solution matic ϕ of $x' = A(t)x, t \in I$ on I is a fundamental matrix of IVP on I iff (a) $\det\phi(t) = 0, t \in I$ (b) $\det\phi(t) = 1, t \in I$ (c) $\det\phi(t) \neq 0, t \in I$ (d) $\det\phi(t) \neq 1, t \in I$	K1	CO2
4.	The Solution of the IVP $x'' - 2x_0' + x = 0; x(0) = 0, x'(0) = 1$ is — (a) $x(t) = e^t, t \in I$ (b) $x(t) = e^{-t}, t \in I$ (c) $x(t) = -te^{-t}, t \in I$ (d) $x(t) = te^t, t \in I$	K2	CO2
5.	Let ϕ denote fundamental matrix for $x' = A(t)x$ Then $\phi(t + \omega), (-\infty < t < \infty)$ is — (a) Fundamental matrix (b) Exporential matrix (c) Square matrix (d) None of the above	K1	CO3
6.	Consider the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then e^{At} is (a) $\begin{bmatrix} \cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$ (b) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (c) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$ (d) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$	K2	CO3
7.	A function $f(t, x)$ satisfies Lipschitz condition if their exists a constant $k > 0, (t, x_1), (t, x_2) \in D$ and $x_1 \neq x_2$, then (a) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } \leq K$ (b) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } < K$ (c) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } \geq K$ (d) $\frac{ f(t, x_1) - f(t, x_2) }{ x_1 - x_2 } > K$	K1	CO4

Cont....

8.	Consider the IVP $x' = x^2, x(0) = 1$. The equation is equivalent to (a) $x(t) = 1 - \int_{t_0}^t x^2(s)ds$ (b) $x(t) = 1 + \int_{t_0}^t x^2(s)ds$ (c) $x(t) = 1 - \int_{t_0}^t x(s)ds$ (d) $x(t) = 1 + \int_{t_0}^t x(s)ds$	K2	CO4
9.	A linear BVP which is not regular is called — (a) Singular non linear BVP (b) Singular linear BVP (c) Singular non linear IVP (d) Singular linear IVP	K1	CO5
10.	Two distinct functions x and y defined and continuous on $[A, B]$ are said to be orthogonal with respect to a continuous weight function $r(t)$ if (a) $\int_A^B r(s)x(s)y(s)ds \neq 0$ (b) $\int_A^B r(s)x(s)y(s)ds = 0$ (c) $\int_A^B r(s)x(s)y(s)ds = 1$ (d) $\int_A^B r(s)x(s)y(s)ds \neq 1$	K2	CO5

	SECTION B (5 x 7 = 35) (Answer ALL questions)		
11(a).	If $P_n(t)$ and $P_m(t)$ are Legendre Polynomials. Then Prove that $\int_{-1}^1 P_n(t)P_m(t)dt = 0$ if $m \neq n$. OR	K2	CO1
11(b).	Find the regular singular point in the equation $t(t-1)^2(t+3)x'' + t^2x' - (t^2+t-1)x = 0$.	K2	CO1
12(a).	Let $A(t)$ be an $n \times n$ matrix which is continuous on I . Suppose a matrix ϕ satisfies $x' = A(t)x, \quad t \in I$ Then prove that $\det \phi$ satisfies the first order equation $(\det \phi)' = (\text{tr } A)(\det \phi)$. OR	K3	CO2
12(b).	Find the fundamental system of solutions for the system $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; t = [0, 1]$.	K3	CO2
13(a).	Find the fundamental matrix of the linear system $x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix} x$. OR	K2	CO3
13(b).	Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.	K2	CO3

14(a).	Consider the IVP $x' = x^2 + \cos^2 t$, $x(0) = 0$. To determine the largest interval of existence of its solution.	K4	CO4
OR			
14(b).	Prove that the error $x(t) - x_n(t)$ satisfies the estimate $ x(t) - x_n(t) \leq \frac{L(Kh)^{n+1}}{K(n+1)!} e^{Kh}; \quad t \in [t_0, t_0 + h]$	K4	CO4
15(a).	Using appropriate Green's functions. Solve $x'' = e^t$, $x(0) = x(1) = 0$.	K5	CO5
OR			
15(b).	consider the equation $x'' + \lambda x = 0$, $0 \leq t \leq \pi$. Find the eigenvalues and eigenfunctions in the following cases: (i) $x'(0) = x'(\pi) = 0$ (ii) $x(0) = 0$, $x'(\pi) = 0$	K5	CO5

SECTION -C (3 × 10=30) (Answer ANY THREE questions)			
16.	Derive the solutions of Bessel Equation.	K3	CO1
17.	State and prove Existence and Uniqueness theorem.	K4	CO2
18.	Determine e^{tA} for the system $x' = Ax$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$.	K2	CO3
19.	State and prove Picard's theorem.	K2	CO4
20.	Prove that $G(t; s) = \begin{cases} -y(t)z(s)/A & \text{if } t \leq S \\ -y(s)z(t)/A & \text{if } t \geq S \end{cases}$ then $x(t)$ is a solution of $L(x) + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(\epsilon, s) f(s) ds$	K5	CO5

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