

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)**

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

DISCRETE MATHEMATICS AND GRAPH THEORY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is an example of a tautology? a) She is both a doctor and a lawyer. b) Either it will rain tomorrow or it will not rain tomorrow. c) The book is on the table. d) He likes pizza more than anything else.	K1	CO1
	2	A ____ statement (or atomic proposition) is a basic statement in logic that cannot be broken down into simpler components. a) primitive b) conditional c) contradict d) normal	K2	CO1
2	3	If $P(n)$ denotes the number of partitions of n , then $P(5) = \underline{\hspace{2cm}}$. a) 12 b) 6 c) 5 d) 4	K1	CO2
	4	The Fibonacci sequence follows the recurrence relation $F(n) = \underline{\hspace{2cm}}$ a) $F(n-1) + F(n-2)$ b) $F(n+1) + F(n-2)$ c) $F(n-1) - F(n-2)$ d) $F(n) + F(n-1)$	K2	CO2
3	5	A ____ relation is a mathematical formula that expresses each term of a sequence in terms of one or more of its previous terms. a) binary b) cubic c) equivalence d) recurrence	K1	CO3
	6	What is the 5th term in the Fibonacci sequence? a) 3 b) 7 c) 8 d) 5	K2	CO3
4	7	Let V be a set of n vertices. The ____ graph on V , denote undirected graph where for all $a, b \in V$, $a \neq b$, there is an edge (a, b) . a) complete graph b) Regular graph c) connectd graph d) Euler graph.	K1	CO4
	8	In a directed graph, what is the in degree of a vertex v ? a) The number of edges leaving vertex v . b) The total number of edges incident to vertex v . c) The number of edges entering vertex v . d) The number of vertices adjacent to vertex v .	K2	CO4
5	9	A tree is a connected graph with no cycles and has n vertices and edges. a) n b) $n+2$ c) $n+1$ d) $n-1$	K1	CO5
	10	What is a spanning tree of a graph? a) A subgraph that contains all vertices of the graph and some of its edges, with no cycles. b) A tree that connects all vertices with the minimum possible edges. c) A graph that includes all edges of the original graph. d) A subgraph that contains all edges of the graph without any vertices.	K2	CO5

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the truth tables for the compound statements $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$.	K2	CO1
		(OR)		
	11.b.	Verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology.		
2	12.a.	How many ways can a police captain distribute 24 refire shells to four police officers so that each officer gets at least three shells, but not more than eight?	K3	CO2
		(OR)		
	12.b.	A company hires 11 new employees, each of whom is to be assigned to 3 one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made?		
3	13.a.	Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ where $n \geq 2$ and $a_0 = 1, a_1 = 2$.	K4	CO3
		(OR)		
	13.b.	Solve the recurrence relation $a_n - 3a_{n-1} = 5(3^n)$, where $n \geq 1$ and $a_0 = 2$.		
4	14.a.	If $G=(V,E)$ is an undirected graph or multigraph, then prove that $\sum_{v \in V} \deg(v) = 2 E $.	K3	CO4
		(OR)		
	14.b.	State and prove decomposition theorem for Chromatic polynomials.		
5	15.a.	For any tree $T=(V,E)$, $ E \geq 2$, Then prove that T has at least two pendant vertices.	K4	CO5
		(OR)		
	15.b.	If T is an optimal tree for the n weights $w_1 \leq w_2 \leq \dots \leq w_n$ then prove that there exist an optimal tree T' in which the leaves of weights w_1 and w_2 are siblings at the maximal level (in T')		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find the d.n.f. for $f:B^3 \rightarrow B$, where $f(x,y,z) = xy + \bar{x}z$.	K3	CO1
2	17	Find a formula to express $0^2 + 1^2 + 2^2 + \dots + n^2$ as a function of n.	K3	CO2
3	18	Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.	K4	CO3
4	19	Let $G = (V,E)$ be a connected planar graph or multigraph with $ V =v$ and $ E =e$. Let r be the number of regions in the plane determined by a planar embedding of G ; one of these regions has infinite area and is called the infinite region. Then prove that $v - e + r = 2$.	K4	CO4
5	20	If $G = (V, E)$ is an undirected graph. Then prove that G is connected if and only if G has a spanning tree.	K3	CO5