

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	A plane has _____ degrees of freedom. a) 2 b) 3 c) 4 d) 0	K1	CO2
2	If the equations of two planes differ only in the constant term, they are _____. a) parallel b) perpendicular c) identical d) intersecting but not perpendicular	K2	CO2
3	The line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$ if $\theta =$ _____. a) 90° b) 45° c) 30° d) 0°	K1	CO1
4	If the two lines are coplanar, they _____. a) never intersect b) are skew c) must intersect d) are parallel	K2	CO1
5	When the centre of the sphere is at the origin and its radius is a, the equation of the sphere is _____. a) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = a$ b) $x^2 + y^2 + z^2 - 2ux - 2vy - 2wz + d = a$ c) $x^2 + y^2 + z^2 = a$ d) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$	K1	CO4
6	The equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ at the point (x_1, y_1, z_1) is _____. a) $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = r^2$ b) $xx_1 + yy_1 + zz_1 = r^2$ c) $x_1^2 + y_1^2 + z_1^2 = r^2$ d) $(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2$	K2	CO4
7	If F is a vector field, then $\text{div } F =$ _____. a) ∇F b) $\nabla \times F$ c) $\nabla \cdot F$ d) $(\nabla \times F)^2$	K1	CO4
8	If F is conservative, then $\text{Curl } F =$ _____. a) 1 b) 0 c) ∇F d) $\nabla \cdot F$	K2	CO4
9	If F is a continuous vector field defined on an oriented surface S with unit normal vector n, then the surface integral of F over S is _____. a) $\int \int_S F \cdot dS = \int \int_S F \cdot dr$ b) $\int \int_S F \cdot dS = \int \int_S F \times dS$ c) $\int \int_S F \cdot dS = \int \int_S F dS$ d) $\int \int_S F \cdot dS = \int \int_S F \cdot dA$	K1	CO5
10	If E be a simple solid region and S be the boundary surface of E. If F be a vector field, then by Divergence theorem $\int \int \int_E \text{div } F \, dV =$ _____. a) $\int \int_S F \cdot dS$ b) $\int \int_S F \times dS$ c) $\int \int_S F \, dS$ d) $\int \int \int_E F \cdot dV$	K2	CO5

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a	Find the Equation of the plane passing through the point $(3, 1, 2), (3, 4, 4)$ and perpendicular to the plane $5x + y + 4z = 0$.	K2	CO1
	(OR)			
	11.b.	Find the equation of the plane through the point $(1, -2, 3)$ and the intersection of the plane $2x - y + 4z = 7$ and $x + 2y - 3 + 8 = 0$.		
2	12.a.	Find the perpendicular distance from $P(3, 9, -1)$ to the line $\frac{x+8}{-8} = \frac{y-31}{1} = \frac{z-13}{5}$.	K4	CO2
	(OR)			
	12.b.	Find the volume of the tetrahedron whose vertices are $(-1, 4, 9); (4, -2, 3); (5, 4, -3)$ and $(1, 3, 1)$.		
3	13.a.	A sphere of constant radius k passes through the origin and meets the axes in A, B, C . Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.	K3	CO4
	(OR)			
	13.b.	Find the equation of a sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 12z + 1 = 0$ at the point $(2, -2, 1)$ and passes through the origin.		
4	14.a.	Find curl and divergence of the vector field (i) $F(x, y, z) = xyz \vec{i} - x^2 y \vec{k}$ (ii) $F(x, y, z) = \vec{i} + (x + yz) \vec{j} + (xy - \sqrt{z}) \vec{k}$	K3	CO2
	(OR)			
	14.b.	Evaluate $\int_C 2x \, dS$, where C consists of the arc C_1 of the parabola $y = x^2$ from $(0, 1)$ to $(1, 1)$ followed by the vertical line segment C_2 from $(1, 1)$ to $(1, 2)$.		
5	15.a.	Use Stoke's Theorem to compute the integral $\int \int_S \text{curl } F \cdot dS$, where $F(x, y, z) = xz \vec{i} + yz \vec{j} + xy \vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane.	K4	CO5
	(OR)			
	15.b.	Evaluate $\int \int_S F \cdot ds$, where $F(x, y, z) = y \vec{i} + x \vec{j} + z \vec{k}$ and S is the boundary of the solid region E enclosed by the parabola $z = 1 - x^2 - y^2$ and the plane $z = 0$.		

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SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Show that the origin lies in the acute angle between the planes $x + 2y + 2z = 0$, $4x - 3y + 12z + 13 = 0$. Find the planes bisecting the angle between them and point out which bisects the obtuse angle.	K4	CO1
17	Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plane through them.	K4	CO1
18	Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$; $x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.	K4	CO4
19	Evaluate $\int_C x^4 dx + xy dy$, where C is the triangular curve consisting of the line segment from $(0, 1)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$.	K4	CO4
20	Verify that the Divergence Theorem is true for the vector field F on the region E : $F(x, y, z) = 3x\vec{i} + xy\vec{j} + 2xz\vec{k}$, E is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.	K4	CO5

Z-Z-Z END