

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)**

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED MATHEMATICAL STATISTICS - I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (1/0 Marks)

Answer **ALL** questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The total number of possible outcomes of a random experiment is known as the _____ events. a) exhaustive b) mutually exclusive c) independent d) equally likely	K1	CO1
	2	A, B and C are three arbitrary events, find the expression that none occurs in terms of A, B and C is _____. a) $A \cup \bar{B} \cup \bar{C}$ b) $\bar{A} \cap \bar{B} \cap \bar{C}$ c) $A \cap B \cap C$ d) $\bar{A} \cap B \cap \bar{C}$	K2	CO1
2	3	The probability density function $p(x)$ cannot exceed _____. a) 0 b) 1 c) ∞ d) $-\infty$	K1	CO2
	4	The conditional probability mass function $p_{Y X}(y x)$ is defined as _____. a) $\frac{P(X=x, Y=y)}{P(X=x)}$ b) $\frac{P(X=x, Y=y)}{P(Y=y)}$ c) $\frac{P(X=x, Y=y)}{P(Y=y)}$ d) $\frac{P(Y=y)}{P(X=x)}$	K2	CO2
3	5	If X and Y are two random variables, the covariance between the variable $aX + b$ and $cY + d$ in terms of $COV(X, Y)$ is _____. a) $aCov(X, Y)$ b) $cdCov(X, Y)$ c) $bdCov(X, Y)$ d) $acCov(X, Y)$	K1	CO3
	6	If X and Y are two random variables such that $Y \leq X$ then _____. a) $E(Y) \geq E(X)$ b) $E(Y) + E(X) = 0$ c) $E(X) \leq E(Y)$ d) $E(X) - E(Y) = 0$	K2	CO3
4	7	M.G.F of the characteristic function $\phi(t)$, also generates moments then _____. a) $\sum_{r=0}^{\infty} \frac{(it)^r}{r!} \mu'_r$ b) $\sum_{r=0}^{\infty} \frac{(it)^r}{r} \mu'_r$ c) $\frac{(it)^r}{r!} \mu'_r$ d) $\sum_{r=0}^{\infty} \frac{(it)^r}{r} \mu_r$	K1	CO4
	8	A random variable X may have no moments although its M.G.F is _____. a) not exist b) 1 c) 0 d) exist	K2	CO4
5	9	The characteristic function of Binomial distribution is _____. a) $(q + pe^{it})^{-n}$ b) $(q + pe^t)^n$ c) $(q + pe^{it})^n$ d) $(q + e^{it})^{-n}$	K1	CO5

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5	10	In a normal distribution, the mean deviation about the mean is approximately equal to _____. (a) $\frac{4}{5}\sigma^2$ b) $\frac{4}{5}\sigma$ c) $\frac{4}{3}\sigma^2$ d) $\frac{5}{4}\sigma$	K2	CO5
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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO																		
1	11.a.	State and prove Addition theorem on probability.	K5/K4	CO1																		
	(OR)																					
	11.b.	From a city population, the probability of selecting (i) a male or a smoker is 7/10,(ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2/3.find the probability of selecting (a) a non-smoker,(b) a male and (c) a smoker, if a male first selected.																				
2	12.a.	A random variable X has the following probability function: <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>$p(x)$</td><td>0</td><td>K</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr></table> (i) Find k (ii) Evaluate $P(X < 6, P(X \geq 6)$ and $P(0 < X < 5)$ (iii) Determine the distribution function of X .	x	0	1	2	3	4	5	6	7	$p(x)$	0	K	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$	K4/K2	CO2
	x	0	1	2	3	4	5	6	7													
	$p(x)$	0	K	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$													
(OR)																						
12.b.	State the properties of continuous distribution function.																					
3	13.a.	(a)Obtain the additive property of mathematical expectation. (b) For any variates X and Y , show that $\{E(X + Y)^2\}^{1/2} \leq \{E(X^2)\}^{1/2} + \{E(Y^2)\}^{1/2}$	K3/K5	CO3																		
	(OR)																					
	13.b.	Explain condition expectation and conditional variance.																				
4	14.a.	Define characteristic function of random variable and prove that if the distribution function of a random variable X is symmetrical about zero then $\phi_X(t)$ is real valued function and even function of t .	K5/K3	CO4																		
	(OR)																					
	14.b.	State uniqueness theorem for moment generating function and if X is a random variable X assume the value ' r ' with the probability law: $P(X = r) = q^{r-1}p; r = 1, 2, 3 \dots$ then (i) find the $m. g. f$ of X and (ii) mean and variance of X																				
5	15.a.	Find the m.g.f about mean of binomial distribution and hence find mean and variance of binomial distribution.	K3/K4	CO5																		

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5	(OR)		K3/K4	CO5
	15.b.	A manufacturer, who produces medicine bottles, find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Find how many boxes will contain: (i) no defective (ii) atleast two defective. ($e^{-0.5} = 0.6065$)		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and Prove Baye's theorem.	K5	CO1
2	17	The probability density function of a two-dimensional random variable (X, Y) is given by: $f(x, y) = \begin{cases} 2; & 0 < x < 1; 0 < y < x; \\ 0, & \text{elsewhere} \end{cases}$ (i) Find the marginal density function of X and Y. (ii) Find the conditional density function of Y given $X = x$ and conditional density function of X given $Y = y$. (iii) check for independence of X and Y.	K4	CO2
3	18	Let $f(x, y) = 8xy, 0 < x < y < 1; f(x, y) = 0$ elsewhere. Find (a) $E(Y X = x)$, (b) $E(XY X = x)$, (c) $Var(Y X = x)$.	K4	CO3
4	19	(a) Prove that if X is some random variable with characteristic function $\phi_X(t)$, and if $\mu'_r = E(X^r)$ exist then $\mu'_r = (-i)^r \left \frac{\partial^r}{\partial t^r} \phi(t) \right _{t=0}$ (b) Show that if $\phi_X(s) = 1$ for some $s \neq 0$, then for some real a, $X - a$ is a lattice variable with mesh, $h = 2\pi/ s$.	K5	CO4
5	20	Obtain the moments of the Normal distribution	K3	CO5

Z-Z-Z

END