

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

SEQUENCE, SERIES AND TRIGONOMETRY

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

- 1 If a sequence diverges to infinity, the sequence is
(i) bounded (ii) bounded above
(iii) bounded below (iv) oscillates
- 2 Consider the statements:
(a) Any bounded sequence is Cauchy
(b) A subsequence of a Cauchy sequence is Cauchy
In the above statements
(i) Both (a) and (b) are true (ii) (a) is true and (b) is false
(iii) (a) is false and (b) is true (iv) Both (a) and (b) are false
- 3 _____ is not absolutely convergent sequence.
(i) $\sum \frac{(-1)^n}{n^2}$ (ii) $\sum \frac{(-1)^n}{n}$
(iii) $\sum \frac{x^{n-1}}{(n-1)!}$ (iv) $\sum \frac{(-1)^n \sin n\alpha}{n^3}$
- 4 $\cosh 2x + \sinh 2x =$ _____
(i) $\frac{1 + \tanh x}{1 - \tanh x}$ (ii) $\frac{1 + \cosh^2 x}{1 - \cosh^2 x}$
(iii) $\frac{1 + \tanh x}{1 - \tanh x}$ (iv) $\frac{1 + \sinh^2 x}{1 - \sinh^2 x}$
- 5 What is the sum of the series $\left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \dots$?
(i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{2}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{\pi}{12}$

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 3 = 15)

- 6 a If the sequence of real numbers $(s_n)_{n=1}^{\infty}$ converges to L then prove that any subsequence of $(s_n)_{n=1}^{\infty}$ is also converges to L .
OR
b If the sequence of real numbers $(s_n)_{n=1}^{\infty}$ is convergent then prove that $(s_n)_{n=1}^{\infty}$ is bounded.
- 7 a Prove that if $(s_n)_{n=1}^{\infty}$ is a Cauchy sequence of real numbers then $(s_n)_{n=1}^{\infty}$ is bounded.
OR
b If $(s_n)_{n=1}^{\infty}$ is a sequence of real numbers which converges to L , then prove that $(s_n^2)_{n=1}^{\infty}$ converges to L^2

Cont....

- 8 a If $\sum_{n=1}^{\infty} a_n$ converges absolutely then prove that $\sum_{n=1}^{\infty} a_n$ converges.

OR

- b Using root test to find the convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

- 9 a Expand $\sin^6 \theta$ in a series of sines of multiples of θ .

OR

- b If $\cos \alpha \cdot \cosh \beta = \cos \phi$, $\sin \alpha \sinh \beta = \sin \phi$ prove that $\sin \phi = \pm \sin^2 \alpha = \pm \sinh^2 \beta$.

- 10 a Sum to infinity the series $c \cos \alpha - \frac{c^3}{3} \cos 3\alpha + \frac{c^5}{5} \cos 5\alpha - \dots$

OR

- b If $\log \sin(\theta + i\phi) = L + iB$, then compute $2e^{2L}$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a Prove that the sequence is $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}_{n=1}^{\infty}$ is convergent.

OR

- b If $(s_n)_{n=1}^{\infty}$ is a sequence of non negative numbers n if $\lim_{n \rightarrow \infty} S_n = L$ then prove that $L \geq 0$.

- 12 a If $(s_n)_{n=1}^{\infty}$ and $(t_n)_{n=1}^{\infty}$ are sequences of real numbers, If $\lim_{n \rightarrow \infty} s_n = L$ and if $\lim_{n \rightarrow \infty} t_n = M$ where $M \neq 0$ then prove that $\lim_{n \rightarrow \infty} \left(\frac{s_n}{t_n} \right) = \frac{L}{M}$.

OR

- b If $(s_n)_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $(s_n)_{n=1}^{\infty}$ is convergent.

- 13 a State and prove Minkowski's inequality.

OR

- b Prove that (i) The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is converges

(ii) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is diverges (iii) The series $\sum_{n=1}^{\infty} \frac{1}{n \log n}$ is diverges

- 14 a Separate into real and imaginary parts $\tan^{-1}(x + iy)$.

OR

- b Analyze $\frac{\sin 7\theta}{\sin \theta} = 7 - 56 \sin^2 \theta + 112 \sin^4 \theta - 64 \sin^6 \theta$.

- 15 a Examine the sum to infinity the series

$$\cos \alpha + \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \frac{3}{4} \cos(\alpha + 2\beta) + \dots$$

OR

- b Find the sum to infinity of the series

$$\sin \alpha + c \sin(\alpha + \beta) + \frac{c^2}{2} \sin(\alpha + 2\beta) + \dots \text{when } |c| < 1$$

Z-Z-Z

END