

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	The _____ of equation is the greatest exponent of the highest order. (a) form (b) order (c) degree (d) linear equation	K1	CO1
2	An equation which is not linear is called a _____ differential equation. (a) linear (b) non-linear (c) fractional (d) constant	K1	CO1
3	A _____ of a differential equation is a relation between the variables, by means of which and the derivatives obtained there from the equation is satisfied. (a) solution (b) complete integral (c) particular integral (d) fraction	K1	CO2
4	Charpit's method is primarily used to find _____. (a) second-order PDEs (b) particular solutions to nonlinear PDEs (c) first-order linear PDEs (d) linear equations	K2	CO2
5	An equation is said to be of order _____, if it involves atleast one of the differential coefficients $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$. (a) zero (b) one (c) two (d) three	K2	CO3
6	The general form of a second-order PDE with variable coefficients includes _____. (a) only constant coefficients (b) variable functions multiplying the derivatives (c) no derivatives (d) only one dependent variable	K2	CO3
7	_____ are infinite series that represent periodic functions terms of cosines and sines. (a) Series (b) Fourier Series (c) Analytic (d) Equations	K1	CO4
8	f(x) is an _____ function if $f(-x) = f(x)$. (a) even (b) odd (c) cosine (d) sine	K2	CO4
9	The one-dimensional wave equation is given by _____. (a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (c) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ (d) $\frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2}$	K2	CO5
10	The values $\lambda_n = \frac{c_n \pi}{L}$ are called the _____ of the vibrating string. (a) function (b) eigen function (c) spectrum (d) eigen values	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Find the partial differential equation by eliminating a and b from the equation $z = ax + by + a^2 + b^2$.	K2	CO1
	(OR)		

Cont...

11.b.	Solve $(y^2z/x)p + xzq = y^2$.	K2	CO1
12.a.	Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.	K3	CO2
(OR)			
12.b.	Find the complete integral of $px + qy = pq$.		
13.a.	Solve $r = 6x$.	K3	CO3
(OR)			
13.b.	Solve $xyz = 1$.		
14.a.	Find the Fourier series of the function $f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$ $p = 2L = 4, L = 2$.	K3	CO4
(OR)			
14.b.	Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.		
15.a.	Prove that $u_n(x, t) = (B_n \cos \lambda_n t + B_n^* \sin \lambda_n t) \sin \frac{n\pi}{L} x$.	K3	CO5
(OR)			
15.b.	Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < L/2 \\ L - x & \text{if } L/2 < x < L \end{cases}$		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Question No.	Question	K Level	CO
16	Form a partial differential equation by eliminating the function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.	K2	CO1
17	Show that the equations $xp - yq = x$ and $x^2 p + q = xz$ are compatible and find their solution.	K3	CO2
18	Solve: $ys - p = xy^2 \cos(xy)$.	K3	CO3
19	Find the Fourier coefficients of the periodic function $f(x)$ if $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.	K3	CO4
20	Prove that $u(x, y) = \sum_{n=1}^{\infty} A_n^* \sin \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}$.	K3	CO5

Z-Z-Z END