PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ADVANCED MATHEMATICAL STATISTICS - I

Time: Three Hours Maximum: 75 Marks

SECTION-A (1/0 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The total number of possible outcomes of a random experiment is known as the _____ events. a) exhaustive b) mutually exclusive c) independent d) equally likely	K1	CO1
	2	A, B and C are three arbitrary events, find the expression that none occurs in terms of A, B and C is _____. a) $A \cup \bar{B} \cup \bar{C}$ b) $\bar{A} \cap \bar{B} \cap \bar{C}$ c) $A \cap B \cap C$ d) $\bar{A} \cap B \cap \bar{C}$	K2	CO1
2	3	The probability density function $p(x)$ cannot exceed _____. a) 0 b) 1 c) ∞ d) $-\infty$	K1	CO2
	4	The conditional probability mass function $p_{Y\|X}(y\|x)$ is defined as _____. a) $\frac{P(X=x,Y=y)}{P(X=x)}$ b) $\frac{P(X=x,Y=y)}{P(Y=x)}$ c) $\frac{P(X=x,Y=y)}{P(Y=y)}$ d) $\frac{P(Y=y)}{P(X=x)}$	K2	CO2
3	5	If X and Y are two random variables, the covariance between the variable $aX + b$ and $cY + d$ in terms of $COV(X,Y)$is _____. a) $aCov(X,Y)$ b) $cdCov(X,Y)$ b) $bdCov(X,Y)$ d) $acCov(X,Y)$	K1	CO3
	6	If X and Y are two random variables such that $Y \leq X$ then _____. a) $E(Y) \geq E(X)$ b) $E(Y) + E(X)$=0 c) $E(X) \leq E(Y)$ d) $E(X) - E(Y) = 0$	K2	CO3
4	7	M.G.F of the characteristic function $\phi(t)$, also generates moments then _____. a) $\sum_{r=0}^{\infty} \frac{(it)^r}{r!} \mu_r'$ b) $\sum_{r=0}^{\infty} \frac{(it)^r}{r} \mu_r'$ c) $\frac{(it)^r}{r!} \mu_r'$ d) $\sum_{r=0}^{\infty} \frac{(it)^r}{r} \mu_r$	K1	CO4
	8	A random variable X may have no moments although its M.G.F is _____. a) not exist b) 1 c) 0 d) exist	K2	CO4
5	9	The characteristic function of Binomial distribution is _____. a) $(q + pe^{it})^{-n}$ b) $(q + pe^{t})^{n}$ c) $(q + pe^{it})^{n}$ d) $(q + e^{it})^{-n}$	K1	CO5

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5	10	In a normal distribution, the mean deviation about the mean is approximately equal to_____. (a) $\frac{4}{5}\sigma^2$ b) $\frac{4}{5}\sigma$ c) $\frac{4}{3}\sigma^2$ d) $\frac{5}{4}\sigma$	K2	CO5

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove Addition theorem on probability.	K5/K4	CO1
		(OR)		
	11.b.	From a city population, the probability of selecting (i) a male or a smoker is 7/10,(ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2/3.find the probability of selecting (a) a non-smoker,(b) a male and (c) a smoker, if a male first selected.		
2	12.a.	A random variable X has the following probability function: x: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 $p(x)$: 0 | K | $2k$ | $2k$ | $3k$ | k^2 | $2k^2$ | $7k^2+k$ (i) Find k (ii) Evaluate $P(X<6, P(X\geq 6)$ and $P(0<X<5)$ (iii) Determine the distribution function of X.	K4/K2	CO2
		(OR)		
	12.b.	State the properties of continuous distribution function.		
3	13.a.	(a)Obtain the additive property of mathematical expectation. (b) For any variates $X$and Y, show that $\{E(X+Y)^2\}^{1/2} \leq \{E(X^2)\}^{1/2} + \{E(Y^2)\}^{1/2}$	K3/K5	CO3
		(OR)		
	13.b.	Explain condition expectation and conditional variance.		
4	14.a.	Define characteristic function of random variable and prove that if the distribution function of a random variable X is symmetrical about zero then $\phi_X(t)$ is real valued function and even function of t.	K5/K3	CO4
		(OR)		
	14.b.	State uniqueness theorem for moment generating function and if$X$ is a random variable X assume the value $'r'$ with the probability law: $P(X=r) = q^{r-1}p; r = 1,2,3$ ...then (i) find the $m.g.f$ of X and (ii) mean and variance of X		
5	15.a.	Find the m.g.f about mean of binomial distribution and hence find mean and variance of binomial distribution.	K3/K4	CO5

5		(OR)	K3/K4	CO5
	15.b.	A manufacturer, who produces medicine bottles, find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Find how many boxes will contain: (i) no defective (ii) atleast two defective. $(e^{-0.5} = 0.6065)$		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	State and Prove Baye's theorem.	K5	CO1
2	17	The probability density function of a two-dimensional random variable (X, Y) is given by: $f(x,y) = \begin{cases} 2; 0 < x < 1; 0 < y < x; \\ 0, \quad elsewhere \end{cases}$ (i) Find the marginal density function of X and Y. (ii) Find the conditional density function of Y given $X = x$ and conditional density function of X given $Y = y$. (iii) check for independence of X and Y.	K4	CO2
3	18	Let $f(x,y) = 8xy, 0 < x < y < 1; f(x,y) = 0$ elsewhere. Find (a)$E(Y\|X = x)$,(b) $E(XY\|X = x)$,(c) $Var(Y\|X = x)$.	K4	CO3
4	19	(a) Prove that if X is some random variable with characteristic function $\phi_X(t)$, and if $\mu_r' = E(X^r)$ exist then $\mu_r' = (-i)^r \left\| \frac{\partial^r}{\partial t^r} \phi(t) \right\|_{t=0}$ (b) Show that if $\|\phi_X(s)\| = 1$ for some $s \neq 0$, then for some real a, $X - a$ is a lattice variable with mesh, $h = 2\pi/\|s\|$.	K5	CO4
5	20	Obtain the moments of the Normal distribution	K3	CO5

Z-Z-Z END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	A plane has _____ degrees of freedom. a) 2 b) 3 c) 4 d) 0	K1	CO2
2	If the equations of two planes differ only in the constant term, they are _____. a) parallel b) perpendicular c) identical d) intersecting but not perpendicular	K2	CO2
3	The line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$ if $\theta =$ _____. a) 90° b) 45° c) 30° d) 0°	K1	CO1
4	If the two lines are coplanar, they _____. a) never intersect b) are skew c) must intersect d) are parallel	K2	CO1
5	When the centre of the sphere is at the origin and its radius is a, the equation of the sphere is _____. a) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = a$ b) $x^2 + y^2 + z^2 - 2ux - 2vy - 2wz + d = a$ c) $x^2 + y^2 + z^2 = a$ d) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$	K1	CO4
6	The equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ at the point (x_1, y_1, z_1) is _____. a) $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = r^2$ b) $xx_1 + yy_1 + zz_1 = r^2$ c) $x_1^2 + y_1^2 + z_1^2 = r^2$ d) $(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2$	K2	CO4
7	If F is a vector field, then div F = _____. a) ∇F b) $\nabla \times F$ c) $\nabla \cdot F$ d) $(\nabla \times F)^2$	K1	CO4
8	If F is conservative, then Curl F = ___. a) 1 b) 0 c) ∇F d) $\nabla \cdot F$	K2	CO4
9	If F is a continuous vector field defined on an oriented surface S with unit normal vector n, then the surface integral of F over S is _____. a) $\int\int_S F.dS = \int\int_S F.dr$ b) $\int\int_S F.dS = \int\int_S F \times dS$ c) $\int\int_S F.dS = \int\int_S \lvert F\rvert dS$ d) $\int\int_S F.dS = \int\int_S F.dA$	K1	CO5
10	If E be a simple solid region and S be the boundary surface of E. If F be a vector field, then by Divergence theorem $\int\int\int_E \text{div } F\, dV =$ _____. a) $\int\int_S F.dS$ b) $\int\int_S F \times dS$ c) $\int\int_S \lvert F\rvert\, dS$ d) $\int\int\int_E F.dV$	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a	Find the Equation of the plane passing through the point $(3,1,2),(3,4,4)$ and perpendicular to the plane $5x+y+4z=0$.	K2	CO1
		(OR)		
	11.b.	Find the equation of the plane through the point $(1,-2,3)$ and the intersection of the plane $2x-y+4z=7$and $x+2y-3+8=0$.		
2	12.a.	Find the perpendicular distance from $P(3,9,-1)$ to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$.	K4	CO2
		(OR)		
	12.b.	Find the volume of the tetrahedron whose vertices are $(-1,4,9);(4,-2,3);(5,4,-3)$and $(1,3,1)$.		
3	13.a.	A sphere of constant radius k passes through the origin and meets the axes in A,B,C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.	K3	CO4
		(OR)		
	13.b.	Find the equation of a sphere which touches the sphere $x^2+y^2+z^2-6x+12z+1=0$ at the point $(2,-2,1)$ and passes through the origin.		
4	14.a.	Find curl and divergence of the vector field (i) $F(x,y,z)=xyz\,\vec{\imath}-x^2y\vec{k}$ (ii) $F(x,y,z)=\vec{\imath}+(x+yz)\vec{\jmath}+(xy-\sqrt{z})\vec{k}$	K3	CO2
		(OR)		
	14.b.	Evaluate $\int_C 2x\,dS$, where C consists of the arc C_1 of the parabola $y=x^2$ from $(0,1)$ to $(1,1)$ followed by the vertical line segment C_2 from $(1,1)$ to $(1,2)$.		
5	15.a.	Use Stoke's Theorem to compute the integral $\int\int_S curl\,F.dS$, where $F(x,y,z)=xz\,\vec{\imath}+yz\,\vec{\jmath}+xy\,\vec{k}$and S is the part of the sphere $x^2+y^2+z^2=4$ that lies inside the cylinder $x^2+y^2=1$ and above the $xy-$plane.	K4	CO5
		(OR)		
	15.b.	Evaluate $\int\int_S F.ds$, where $F(x,y,z)=y\,\vec{\imath}+x\,\vec{\jmath}+z\,\vec{k}$and S is the boundary of the solid region E enclosed by the parabola $z=1-x^2-y^2$ and the plane $z=0$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Show that the origin lies in the acute angle between the planes $x + 2y + 2z = 0, 4x - 3y + 12z + 13 = 0$. Find the planes bisecting the angle between them and point out which bisects the obtuse angle.	K4	CO1
17	Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plane through them.	K4	CO1
18	Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0; x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.	K4	CO4
19	Evaluate $\int_C x^4\,dx + xy\,dy$, where C is the triangular curve consisting of the line segment from $(0,1)$ to $(1,0)$, from $(1,0)$ to $(0,1)$ and from $(0,1)$ to $(0,0)$.	K4	CO4
20	Verify that the Divergence Theorem is true for the vector field F on the region E: $F(x,y,z) = 3x\,\vec{i} + xy\,\vec{j} + 2xz\,\vec{k}$, E is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.	K4	CO5

Z-Z-Z END

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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The order of the equation is $y^{(4)} + x^2y^{(3)} + x^5y = sinx$ is a)Three b) Five c)Four d) Six	K1	CO1
	2	A fruit fly population in a closed container is the classical example of a _____ a)Competition situation b) Joint proportion situation c)Limited environment situation d) None	K2	CO1
2	3	The equation: $y'' = yy'$ is a)Linear second order equation b) Non linear second order equation c) Homogenous equation d)Non homogenous equation	K1	CO1
	4	The general solution of $2y'' - 7y' + 3y = 0$ is a)$c_1e^x + c_2e^{3x}$ b)$c_1e^{x/2} + c_2e^{3x}$ c)$c_1e^x + c_2e^{\frac{3}{2}x}$ d) $c_1e^{x/2} + c_2e^{\frac{3}{2}x}$	K2	CO1
3	5	An Euler's formula is a)$e^{i\theta} = cos\theta - isin\theta$ b)$e^{-i\theta} = cos\theta + isin\theta$ c) $e^{i\theta} = cos\theta + isin\theta$ d)$e^{i\theta} = sin\theta + icos\theta$	K1	CO1
	6	Variation of parameters method is used to find a _____ of the non-homogenous linear differential equation. a)General solution b) Initial solution c)Particular solution d) Complimentary solution	K2	CO1
4	7	The value of : $\Gamma\left(\frac{5}{2}\right)$ = _____ a)$\frac{3}{2}\sqrt{\pi}$ b) $\frac{4}{3}\sqrt{\pi}$ c) $\frac{3}{4}\sqrt{\pi}$ d) $\frac{2}{3}\sqrt{\pi}$	K1	CO1
	8	$L(e^{at}t^n) =$ a)$\frac{n!}{(s+a)^{n+1}}$ b) $\frac{n!}{(s-a)^{n-1}}$ c) $\frac{n!}{(s-a)^{n+1}}$ d) $\frac{n!}{(s+a)^{n-1}}$	K2	CO1
5	9	$L^{-1}\left\{tan^{-1}\frac{1}{s}\right\} =$ a)$\frac{cost}{t}$ b) $\frac{sint}{t}$ c)$tcost$ d)$tsint$	K1	CO1

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5	10	The least positive value of p for which f (t+p)= f (t) hold is called the _____ of f a)Transform b) Period c)Laplace value d) Inverse Laplace value	K2	CO1

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Verify that the function : $y(x) = 2x^{1/2} - x^{1/2}lnx$ satisfies the differential equation $4x^2y'' + y = 0$ for all x>0.	K3	CO2
		(OR)		
	11.b.	Solve the initial value problem $\frac{dy}{dx} - y = \frac{11}{8}e^{-x/3}$, y(0) = -1.		
2	12.a.	Derive the principle of super position for homogenous equations.	K3	CO2
		(OR)		
	12.b.	Find a solution of $y'' + 4y = 12x$ that satisfies the initial conditions $y(0) = 5, y'(0) = 7$		
3	13.a.	Solve the initial value between $y^{(3)} + 3y'' - 10y' = 0$; y(0)=7, $y''(0) = 70$	K3	CO2
		(OR)		
	13.b.	Find a particular solution of $y'' + 4y = 3x^2$.		
4	14.a.	Show that $L(te^{at}) = \frac{1}{(s-a)^2}$.	K3	CO3
		(OR)		
	14.b.	Consider a mass-and-spring system with $m = \frac{1}{2}, k = 17, and\ c = 3$ in mks units. Let x(t) denote the displacement of the mass m from its equilibrium position. If the mass is set in motion with $x(0) = 3$ & $x'(0) = 1$. Find x(t) for the resulting damped free oscillations.		
5	15.a.	Evaluate $L^{-1}\left[\frac{2s}{(s^2-1)^2}\right]$.	K3	CO3
		(OR)		
	15.b.	Let f(t) be periodic with period p and piecewise continuous for $t \geq 0$. Then prove that the transform $F(s) = L\{f(t)\}$ exists for $s > 0$ and is given by $F(s) = \frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)dt$.		

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SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find a general solution of $(x^2+1)\frac{dy}{dx}+3xy=6x.$	K4	CO4
2	17	Solve the initial value problem $y''+2y'+y=0;\ \ y(0)=5,\ y'(0)=-3\ .$	K4	CO4
3	18	Find a particular solution of the equation $y''+y=tanx$ by using variation of parameter method.	K4	CO4
4	19	Solve the initial value problem $y''+4y'+4y=t^2;\ \ y(0)=y'(0)=0$	K4	CO5
5	20	Find (i) $L\{g(t)\}$ if $g(t)=\begin{cases}0 & if\ t<3\\ t^2 & if\ t\geq 3\end{cases}$ (ii)) $L\{f(t)\}$ if $f(t)=\begin{cases}cos2t & if\ 0\leq t<2\pi\\ 0 & if\ t\geq 2\pi\end{cases}$	K4	CO5

Z-Z-Z END