

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**BSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)**

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The order of the equation is $y^{(4)} + x^2y^{(3)} + x^5y = \sin x$ is a) Three b) Five c) Four d) Six	K1	CO1
	2	A fruit fly population in a closed container is the classical example of a _____ a) Competition situation b) Joint proportion situation c) Limited environment situation d) None	K2	CO1
2	3	The equation: $y'' = yy'$ is a) Linear second order equation b) Non linear second order equation c) Homogenous equation d) Non homogenous equation	K1	CO1
	4	The general solution of $2y'' - 7y' + 3y = 0$ is a) $c_1e^x + c_2e^{3x}$ b) $c_1e^{x/2} + c_2e^{3x}$ c) $c_1e^x + c_2e^{\frac{3}{2}x}$ d) $c_1e^{x/2} + c_2e^{\frac{3}{2}x}$	K2	CO1
3	5	An Euler's formula is a) $e^{i\theta} = \cos\theta - i\sin\theta$ b) $e^{-i\theta} = \cos\theta + i\sin\theta$ c) $e^{i\theta} = \cos\theta + i\sin\theta$ d) $e^{i\theta} = \sin\theta + i\cos\theta$	K1	CO1
	6	Variation of parameters method is used to find a _____ of the non-homogenous linear differential equation. a) General solution b) Initial solution c) Particular solution d) Complimentary solution	K2	CO1
4	7	The value of : $\Gamma\left(\frac{5}{2}\right) =$ _____ a) $\frac{3}{2}\sqrt{\pi}$ b) $\frac{4}{3}\sqrt{\pi}$ c) $\frac{3}{4}\sqrt{\pi}$ d) $\frac{2}{3}\sqrt{\pi}$	K1	CO1
	8	$L(e^{at}t^n) =$ a) $\frac{n!}{(s+a)^{n+1}}$ b) $\frac{n!}{(s-a)^{n-1}}$ c) $\frac{n!}{(s-a)^{n+1}}$ d) $\frac{n!}{(s+a)^{n-1}}$	K2	CO1
5	9	$L^{-1}\left\{\tan^{-1}\frac{1}{s}\right\} =$ a) $\frac{\cos t}{t}$ b) $\frac{\sin t}{t}$ c) $t\cos t$ d) $t\sin t$	K1	CO1

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5	10	The least positive value of p for which $f(t+p) = f(t)$ hold is called the _____ of f a) Transform b) Period c) Laplace value d) Inverse Laplace value	K2	CO1
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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Verify that the function : $y(x) = 2x^{1/2} - x^{1/2} \ln x$ satisfies the differential equation $4x^2 y'' + y = 0$ for all $x > 0$.	K3	CO2
		(OR)		
	11.b.	Solve the initial value problem $\frac{dy}{dx} - y = \frac{11}{8} e^{-x/3}$, $y(0) = -1$.		
2	12.a.	Derive the principle of super position for homogenous equations.	K3	CO2
		(OR)		
	12.b.	Find a solution of $y'' + 4y = 12x$ that satisfies the initial conditions $y(0) = 5, y'(0) = 7$		
3	13.a.	Solve the initial value between $y^{(3)} + 3y'' - 10y' = 0$; $y(0)=7, y''(0) = 70$	K3	CO2
		(OR)		
	13.b.	Find a particular solution of $y'' + 4y = 3x^2$.		
4	14.a.	Show that $L(te^{at}) = \frac{1}{(s-a)^2}$.	K3	CO3
		(OR)		
	14.b.	Consider a mass-and-spring system with $m = \frac{1}{2}$, $k = 17$, and $c = 3$ in mks units. Let $x(t)$ denote the displacement of the mass m from its equilibrium position. If the mass is set in motion with $x(0) = 3$ & $x'(0) = 1$. Find $x(t)$ for the resulting damped free oscillations.		
5	15.a.	Evaluate $L^{-1} \left[\frac{2s}{(s^2-1)^2} \right]$.	K3	CO3
		(OR)		
	15.b.	Let $f(t)$ be periodic with period p and piecewise continuous for $t \geq 0$. Then prove that the transform $F(s) = L\{f(t)\}$ exists for $s > 0$ and is given by $F(s) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t) dt.$		

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SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find a general solution of $(x^2 + 1) \frac{dy}{dx} + 3xy = 6x.$	K4	CO4
2	17	Solve the initial value problem $y'' + 2y' + y = 0; \quad y(0) = 5, \quad y'(0) = -3.$	K4	CO4
3	18	Find a particular solution of the equation $y'' + y = \tan x$ by using variation of parameter method.	K4	CO4
4	19	Solve the initial value problem $y'' + 4y' + 4y = t^2; \quad y(0) = y'(0) = 0$	K4	CO5
5	20	Find (i) $L\{g(t)\}$ if $g(t) = \begin{cases} 0 & \text{if } t < 3 \\ t^2 & \text{if } t \geq 3 \end{cases}$ (ii) $L\{f(t)\}$ if $f(t) = \begin{cases} \cos 2t & \text{if } 0 \leq t < 2\pi \\ 0 & \text{if } t \geq 2\pi \end{cases}$	K4	CO5

Z-Z-Z END