

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fifth Semester)

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

ABSTRACT ALGEBRA

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

- 1 The relation congruence modulo n defines an equivalence relation on the set of _____.
(i) real numbers (ii) natural numbers
(iii) integers (iv) rational numbers
- 2 Every sub group of an abelian group is -----
(i) normal (ii) an ideal (iii) integral domain (iv) semi-group
- 3 Any group of order p^2 is _____.
(i) normal (ii) abelian (iii) semi-group (iv) isomorphic
- 4 If a commutative ring has no zero divisors then that is called _____.
(i) Ideal (ii) Division ring
(iii) Integral domain (iv) Field
- 5 Let R be a commutative ring with unit element. An element $a \in R$ is a unit in R if there exists an element $b \in R$ such that _____.
(i) $a + b = 1$ (ii) $a - b = 1$ (iii) $\frac{a}{b} = 1$ (iv) $ab = 1$

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a) Prove that a nonempty subset H of a group G is a subgroup of G if and only if $a, b \in H$ implies that $ab \in H$ and $a \in H$ implies that $a^{-1} \in H$.
OR
b) Show that the relation $a \equiv b \pmod{H}$ is an equivalence relation.
- 7 a) Prove that HK is a subgroup of G if and only if $HK = KH$.
OR
b) If G is a finite group and N is a normal subgroup of G then prove that $o(G/N) = o(G)/o(N)$.
- 8 a) Let G be a group and ϕ an automorphism of G . If $a \in G$ is of order $o(a) > 0$ then prove that $o(\phi(a)) = o(a)$.
OR
b) Prove that every permutation is a product of 2-cycles.
- 9 a) If ϕ is a homomorphism of R into R' then prove that $\phi(0) = 0$ and $\phi(-a) = -\phi(a)$ for every $a \in R$.
OR
b) Prove that a finite integral domain is a Field.

Cont...

- 10 a) Let R be a commutative ring with unit element whose only ideals are (0) and R itself then prove that R is a Field .

OR

- b) Let R be a Euclidean ring prove that any two elements a and b in R have a greatest common divisor d . Moreover prove that $d = \lambda a + \mu b$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a) If G is a group then prove the following:
 (i) The identity element of G is unique
 (ii) Every $a \in G$ has a unique inverse in G
 (iii) For every $a \in G$, $(a^{-1})^{-1} = a$
 (iv) For all $a, b \in G$, $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$
 OR
 b) If G is a finite group and H is a subgroup of G then prove that $o(H)$ is a divisor of $o(G)$.
- 12 a) State and prove Cauchy's theorem for abelian groups.
 OR
 b) State and prove Sylow's theorem for abelian groups .
- 13 a) If G is a group then prove that the set of automorphism of G is also a group.
 OR
 b) If G is a group, H a subgroup of G and S is the set of all right cosets of H in G Then prove that there is a homomorphism θ of G into $A(S)$ and the kernel of θ is the largest normal subgroup of G which is contained in H .
- 14 a) If R is a ring, for all $a, b \in R$ then prove the following
 (i) $a \cdot 0 = 0a = 0$
 (ii) $a(-b) = (-a)b = -(ab)$
 (iii) $(-a)(-b) = ab$
 If in addition, R has a unit element 1 then
 (iv) $(-1)a = -a$
 (v) $(-1)(-1) = 1$
 OR
 b) If p is a prime number then prove that J_p the ring of integers mod p is a field .
- 15 a) Prove that $J[i]$ is a Euclidean ring.
 OR
 b) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R .

Z-Z-Z

END