

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)**

Branch - MATHEMATICS

MATHEMATICAL STATISTICS - I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	Given A, B and C are three arbitrary events, then expression for the event at least one occurs is (a) $A \cap B \cap C$ (b) $A \cup B \cup C$ (c) $\overline{A \cap B \cap C}$ (d) $\overline{A \cup B \cup C}$	K1	CO1
2	Given A and B are independent event, then $P(A \cap B)$ is (a) $P(A) \cdot P(B)$ (b) $P(B)$ (c) $P(A) - P(B)$ (d) $P(A) + P(B)$	K2	CO1
3	Give F is a distribution function of a one-dimensional random variable X, then which one is true (a) $F(-\infty) = 1$ (b) $F(\infty) = 0$ (c) $0 \leq F(X) \leq 1$ (d) $F(X) \geq F(Y), x < y$	K1	CO2
4	The mathematical expectation of a random variable X and its square are $\frac{1}{2}, \frac{1}{4}$ respectively, then the variance of X is (a) 1 (b) $\frac{3}{8}$ (c) $\frac{7}{16}$ (d) 0	K2	CO2
5	$M_{cX}(t) = \underline{\hspace{2cm}}$ (a) $cM_X(t)$ (b) $M_X(ct)$ (c) $M_c(Xt)$ (d) none of these	K1	CO3
6	$E[V(X Y)] + V[E(X Y)] = \underline{\hspace{2cm}}$ (a) $E(X Y)$ (b) $V(X Y)$ (c) $E(X)$ (d) $V(X)$	K2	CO3
7	In a Binomial distribution the mean is 4 and the variance is $\frac{8}{3}$. Then the values of n and p are respectively _____ (a) $12, \frac{1}{3}$ (b) $12, \frac{2}{3}$ (c) $16, \frac{1}{3}$ (d) $12, \frac{1}{9}$	K1	CO4
8	The quartile deviation of a normal distribution is approximately _____ (a) $\frac{2\sigma}{\sqrt{3}}$ (b) $\frac{2\sigma}{3}$ (c) $\frac{3\sigma}{\sqrt{2}}$ (d) $\frac{4\sigma}{5}$	K2	CO4
9	Which one of the following is a random sampling? (a) Tippet's random number (b) throwing a die (c) draw a lottery (d) All of these	K1	CO5
10	The principle of least squares states that the parameters involved in $f(x)$ should be chosen in such a way that $\sum y_i - f(x)$ (a) minimum (b) maximum (c) equal (d) different	K2	CO5

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Question No.	Question	K Level	CO														
11.a.	Two dice are tossed. Find the probability that the sum is (a) greater than 8 (b) neither 7 nor 11.	K2	CO1														
(OR)																	
11.b.	State and prove addition theorem of probability.																
12.a.	The diameter of an electric cable say X, is assumed to be a continuous random variable with p.d.f $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$. (i) Check that $f(x)$ is p.d.f (ii) Determine a number b such that $P(x < b) = P(x > b)$.	K3	CO2														
(OR)																	
12.b.	State and Prove multiplication theorem of expectation and discuss its generalization.																
13.a.	The joint p.d.f of two random variables x and y are given by $P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$ and $P(X = 1, Y = 1) = \frac{1}{3}$. Find (i) marginal distribution of x and y (ii) conditional p.d.f of X given Y= 1.	K2	CO3														
(OR)																	
13.b.	Given $f(x, y) = 8xy$, $0 < x < y < 1$; $f(x, y) = 0$ elsewhere. Find (a) $E(Y X = x)$ (b) $E(XY X = x)$ (c) $Var(Y X = x)$																
14.a.	List out the properties of Normal distribution.	K3	CO4														
(OR)																	
14.b.	Given that $x = 4y + 5$ and $y = kx + 4$ are the regression line of X on Y and Y on X respectively. Show that $0 \leq k \leq \frac{1}{4}$. If $k = \frac{1}{6}$, then find the mean of X and Y. Also find γ_{xy} .																
15.a.	Fit a straight line to the following data <table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td><td>8</td></tr><tr><td>Y</td><td>2.4</td><td>3.0</td><td>3.6</td><td>4</td><td>5</td><td>6</td></tr></table>	X	1	2	3	4	6	8	Y	2.4	3.0	3.6	4	5	6	K3	CO5
X	1	2	3	4	6	8											
Y	2.4	3.0	3.6	4	5	6											
(OR)																	
15.b.	Fit a parabola of second degree to the following data <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Y</td><td>1</td><td>1.8</td><td>1.3</td><td>2.5</td><td>6.3</td></tr></table>	X	0	1	2	3	4	Y	1	1.8	1.3	2.5	6.3				
X	0	1	2	3	4												
Y	1	1.8	1.3	2.5	6.3												

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SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Question No.	Question	K Level	CO																		
16	The contents of Urn – I, II and III are as follows: Urn – I: 1 white, 2 black and 3 red balls Urn – II: 2 white, 1 black and 1 red balls Urn – III: 4 white, 5 black and 3 red balls. One Urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the probability that they come from Urn I, II and III?	K2	CO1																		
17	A random variable X has the following probability function. (i) Find k (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$ (iii) If $P(X \leq a) > \frac{1}{2}$ find the minimum value of a and (iv) Determine the distribution function of X. <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k²</td><td>2k²</td><td>7k² + k</td></tr></table>	x	0	1	2	3	4	5	6	7	P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k	K3	CO2
x	0	1	2	3	4	5	6	7													
P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k													
18	Given that two-dimensional continuous random variable (X, Y) has joint p.d.f given by $f(x, y) = \begin{cases} 6x^2y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ (i) Verify that $\int_0^1 \int_0^1 f(x, y) dx dy = 1$ (ii) Find $P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right)$, $P(X + Y < 1)$, $P(X > Y)$ and $P(X < 1 Y < 2)$	K2	CO3																		
19	Fit a Binomial distribution for the following data: <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>Total</td></tr><tr><td>f</td><td>5</td><td>18</td><td>28</td><td>12</td><td>7</td><td>6</td><td>4</td><td>80</td></tr></table>	x	0	1	2	3	4	5	6	Total	f	5	18	28	12	7	6	4	80	K3	CO4
x	0	1	2	3	4	5	6	Total													
f	5	18	28	12	7	6	4	80													
20	Define a Sample. Explain the various methods of random sampling.	K3	CO5																		

Z-Z-Z

END