

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch - MATHEMATICS

CALCULUS – I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$\vec{r}(t) = (1 + t^3)\vec{i} + te^{-t}\vec{j} + \frac{\sin t}{t}\vec{k}$, then $\lim_{t \rightarrow 0} \vec{r}(t) = \underline{\hspace{2cm}}$. a) $\vec{i}$ b) $\vec{i} + \vec{j}$ c) $\vec{i} + \vec{k}$ d) $\vec{i} - \vec{k}$	K1	CO1
	2	$\vec{r}(t) = \langle t, t^2, t^3 \rangle$ is the vector equation of ____. a) helix b) spiral c) knot d) twisted cube	K2	CO1
2	3	If $f(x, y) = x \ln(y^2 - x)$ then $f(3, 2) = \underline{\hspace{2cm}}$. a) 3 b) 0 c) ∞ d) -3	K1	CO2
	4	The equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ is called ____ equation a) Laplace b) Wave c) Euler d) Heat	K2	CO2
3	5	If $f(x, y) = \sin x + e^{xy}$ then $\nabla f(0, 1) = \underline{\hspace{2cm}}$. a) $\langle 0, 0 \rangle$ b) $\langle 2, 2 \rangle$ c) $\langle 0, 2 \rangle$ d) $\langle 2, 0 \rangle$	K1	CO3
	6	If $f(a, b)$ is neither a maximum nor minimum then (a, b) is a ____ point. a) maximum b) minimum c) saddle d) extreme	K2	CO3
4	7	If $f(x, y) \geq g(x, y)$ for all x in R then which of the following property holds: a) $\iint_R f(x, y) dA \geq \iint_R g(x, y) dxdy$ b) $\iint_R f(x, y) dA \leq \iint_R g(x, y) dxdy$ c) $\iint_R f(x, y) dA = \iint_R g(x, y) dxdy$ d) $\iint_R f(x, y) dA = - \iint_R g(x, y) dxdy$	K1	CO4
	8	The mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$ is ____. a) $\frac{11}{16}$ b) $\frac{16}{11}$ c) $\frac{3}{8}$ d) $\frac{8}{3}$	K2	CO4

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5	9	The equation of the surface $z^2 = x^2 + y^2$ in cylindrical coordinates is _____ a) $z = r^2$ b) $z^2 = r$ c) $z = r$ d) $z = \sqrt{r}$	K1	CO5
	10	The cylindrical coordinates of the point with rectangular coordinates $(3, -3, -7)$ is _____. a) $(-3\sqrt{2}, 7\pi/4, 7)$ b) $(3\sqrt{2}, 7\pi/4, -7)$ c) $(3\sqrt{2}, -7\pi/4, 7)$ d) $(-3\sqrt{2}, 7\pi/4, -7)$	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Identify a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.	K3	CO1
	(OR)			
	11.b.	Calculate the curvature of the parabola $y = x^2$ at the points $(0,0)$, $(1,1)$ and $(2,4)$		
2	12.a.	Examine whether $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$ exists if $f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$.	K4	CO2
	(OR)			
	12.b.	a) Inspect if $u(x,y) = e^x \sin y$ satisfies Laplace equation. b) Analyze if $u(x,t) = \sin(x - at)$ satisfies wave equation.		
3	13.a.	The pressure P (in kilopascals), volume V (in liters), and temperature T (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Discuss the rate at which the pressure is changing when the temperature is $300K$ and increasing at a rate of $0.1K/s$ and the volume is $100 L$ and increasing at a rate of $0.2K/s$.	K2	CO3
	(OR)			
	13.b.	Describe the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.		
4	14.a.	Calculate $\iint_D xy \, dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.	K3	CO4
	(OR)			
	14.b.	Apply multiple integrals to find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the xy -plane and inside the cylinder $x^2 + y^2 = 2x$.		

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5	15.a.	Simplify the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx$ in the following order integrating first with respect to x , then z and then y .	K4	CO5
	(OR)			
	15.b.	Deduce the centre of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$; $z = 0$; $x = 1$.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Detect the unit normal and the binormal vectors for the circular helix $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + t \vec{k}$.	K4	CO1
2	17	The dimensions of a rectangular box are measured to be 75cm, 60cm and 40cm and each measurement is correct to within 0.2cm. Use differentials to examine the largest possible error when the volume of the box is calculated from these measurements.	K4	CO2
3	18	Test for the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) / 0 \leq x \leq 3, 0 \leq y \leq 2\}$	K4	CO3
4	19	The density at any point on a semi-circular lamina is proportional to the distance from the centre of the circle. List the coordinates of the centre of mass of the lamina.	K4	CO4
5	20	Simplify the triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$; $x = 2y$; $x = 0$; $z = 0$.	K4	CO5

Z-Z-Z

END