

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2024
(Third Semester)

Branch - **MATHEMATICS**
MECHANICS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	Calculate the magnitude of resultant of two like parallel forces of 20 N separated by a distance of 20 cm. a) 40 N b) 60 N c) 20 N d) 0 N	K1	CO2
2	In case of forces, a couple means _____ a) Two unequal forces acting at two points b) Two equal and like parallel forces acting at two points c) Two equal and perpendicular forces acting at two points d) Two equal and opposite forces acting at two points	K2	CO2
3	CG of a plane lamina is not at its geometrical centre, if it is a: a) Circle b) Square c) Rectangle d) Right angled triangle	K1	CO3
4	_____ is done when a force moves an object through some distance. a) work b) mass c) energy d) force	K2	CO3
5	For the point moving on a straight line which of following is true. a) Has no radial component of acceleration b) Has no tangential component of acceleration c) Has both tangential and radial component of acceleration d) Has no radial and tangential component of acceleration	K1	CO4
6	The rate of change of velocity is known as _____. a) Acceleration b) Speed c) Momentum d) Force	K2	CO4
7	There is an opposite and equal reaction to every action. This situation is described by: a) the first law of motion b) second law of motion c) law of inertia d) third law of motion	K1	CO3
8	What is the force applied on a body with 5 kg of mass and an acceleration of 7 m/s ² ? a) 35 N b) 5 N c) 7 N d) 0 N	K2	CO3
9	A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be _____. a) Vertical straight line b) Inclined straight line c) Parabolic d) Horizontal straight line	K1	CO5
10	On calculating which of the following quantities, the mass of the body has an effect in simple projectile motion? a) Velocity b) Force c) Time of flight d) Range	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Explain the resultant of two like parallel forces acting on a rigid body.	K2	CO2
	(OR)		
11.b.	Prove that theorem on the resultant of a couple and forces.		
12.a.	Develop the Centre of gravity of a symmetry and uniform triangular lamina.	K3	CO4
	(OR)		
12.b.	Identify the Forces which may be omitted in forming the equation of virtual work.		
13.a.	Explain Polygon of force statement and Relate magnitude with direction.	K2	CO3
	(OR)		
13.b.	Interpret the relative angular velocity between two moving points.		
14.a.	Explain Newton's second law of motion.	K3	CO4
	(OR)		
14.b.	Develop the work done in stretching an elastic string.		
15.a.	Deduct the Range on an inclined plane.	K5	CO2
	(OR)		
15.b.	Justify the Theorem of perpendicular axes.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Question No.	Question	K Level	CO
16	State and Prove the Varignon's theorem of moments.	K4	CO4
17	Deduct the Centre of gravity of three rods forming a triangle.	K5	CO3
18	Explain the Angular velocity of any point on a wheel rolling uniformly.	K2	CO2
19	Apply the principle of energy in the case of a freely falling body.	K3	CO3
20	Explain the Fundamental principles and plane path of the projectile with characteristics of the motion of projectile.	K3	CO3

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – MATHEMATICS

MODERN ALGEBRA

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. The two mappings σ and τ of S into T are said to be equal if -----for every $s \in S$.
a) $s\sigma = s\tau$ b) $s\sigma \geq s\tau$ c) $s\sigma \leq s\tau$ d) $s\sigma \neq s\tau$
2. A homomorphism ϕ from G into $\bar{G}$ is said to be an isomorphism if ϕ is one-to-one
a) Automorphism b) Onto c) Isomorphism d) Homomorphism
3. Every permutation is the product of its-----
a) m -cycle. b) Alternating group c) Permutation d) Cycles
4. The homomorphism ϕ of R into R' is an ----- if and only if $I(\phi) = (0)$.
a) Transpositions. b) Permutation c) Isomorphism d) Homomorphism
5. Let R be a -----with unit element. An element $a \in R$ is a unit in R if there exists an element $b \in R$ such that $ab = 1$.
a) Commutative ring . b) Associative ring c) Division ring d) Euclidean ring

SECTION-B (15 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 3 = 15)

- 6 .a) If a and b are integers, not both 0, then (a, b) exists; moreover, we can find integers m_0 and n_0 such that $(a, b) = m_0a + n_0b$.
(OR)
b) Prove that If G is a finite group and $a \in G$, then $o(a) \mid o(G)$.
- 7.a) Prove that If HK is subgroup of G if and only if $HK=KH$.
(OR)
b) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G .
- 8.a) Prove that every permutation is the product of its cycles.
(OR)
b) Prove that let G be a group and ϕ an automorphism of G . If $a \in G$ is of order $o(a) > 0$, then $o(\phi(a)) = o(a)$.

Cont...

9.a) If R is a ring, then for all $a, b \in R$

1. $a0 = 0a = 0$.
2. $a(-b) = (-a)b = -(ab)$.
3. $(-a)(-b) = ab$. If in addition, R has a unit element 1, then
4. $(-1)a = -a$.
5. $(-1)(-1) = 1$.

(OR)

b) Define (i) zero-divisor (ii) integral domain (iii) field

10.a) Prove that let R be a Euclidean ring. Then every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R .

(OR)

b) Prove that let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field.

SECTION - C (30 Marks)

Answer ALL the questions.

ALL questions carry EQUAL marks (5 x 6 = 30)

11.a) Prove that Any positive integer $a > 1$ can be factored in a unique way as

$$a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_t^{\alpha_t} \text{ where } p_1 > p_2 > \dots > p_t \text{ are prime numbers and where each } \alpha_i > 0.$$

(OR)

b) State and prove Lagranges theorem.

12.a) State and prove fundamental theorem of homomorphism.

(OR)

b) State and prove Cauchy's theorem for Abelian groups.

13.a) State and prove Cayley's Theorem.

(OR)

b) Prove that if G is a group, then $\mathcal{A}(G)$, the set of automorphisms of G , is also a group.

14.a) Prove that If p is a prime number then J_p , the ring of integers mod p , is a field.

(OR)

b) Prove that finite integral domain is a field.

15.a) Prove that $J[i]$ is a Euclidean ring.

(OR)

b) Prove that

(i). If p is a prime number of the form $4n + 1$, then $p = a^2 + b^2$ for some integers a, b .

(ii). If p is a prime number of the form $4n + 1$, then we can solve the congruence

$$x^2 = -1 \pmod{p}.$$

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fourth Semester)

Branch – MATHEMATICS

MATHEMATICAL STATISTICS-II

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. If $x_1, x_2, \dots, x_n$ be a random sample from an $N(\mu, \sigma^2)$ population, the sufficient statistics for μ is _____.
(a) $\Sigma(X_i - \bar{x})$ (b) $\frac{\bar{x}}{n}$
(c) $\Sigma(X_i \mu)^2$ (d) none of the above
2. If $X_1, X_2, \dots, X_n$ is a random sample from a population $\frac{1}{\theta\sqrt{2\pi}} e^{-x^2/2\theta^2}$, the maximum likelihood for θ is _____.
(a) $\sqrt{\frac{\Sigma X_i^2}{n}}$ (b) $\frac{\Sigma X_i^2}{n}$ (c) $\sqrt{\frac{\Sigma_i X_i^2}{n}}$ (d) $\frac{\Sigma X_i}{n}$
3. A hypothesis which defines the population distribution is called?
(a) Null Hypothesis (b) Statistical Hypothesis
(c) Simple Hypothesis (d) Composite Hypothesis
4. The range of Level of Significance lies between _____.
(a) $-\infty$ and 0 (b) $-\infty$ and ∞
(c) 0 and ∞ (d) 0 and 1
5. The purpose of blocking in RBD _____.
(a) To reduce instrument error (b) To increase treatment variability.
(c) To avoid randomization. (d) To simply data analysis.

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

6. (a) Find the uniformly minimum variance unbiased estimator for $e^{-\theta}$ based on a random sample drawn from the Poisson (θ) distribution.
(OR)
(b) Write a short note on Consistency.
7. (a) Let $X_1, X_2, \dots, X_n$ is a random sample from $N(\theta, 1)$. Obtain minimum variance of unbiased estimator of θ .
(OR)
(b) Give a brief note on minimum variance with examples.
8. (a) Write a short note on Statistical hypothesis.
(OR)
(b) Elucidate the concept of power of test.

Cont....

9. (a) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease is 85% in favor of the hypothesis that it is more at 5% level?

(OR)

- (b) Find the least value of r in a sample of 18 pairs of observations from a bi-variate normal population, significant at 5% level of significance.
10. (a) Write a short note on complete and incomplete block designs.
- (OR)
- (b) Describe the procedure for complete randomized design.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

11. (a) State and prove Rao-Blackwell theorem.

(OR)

- b) Derive the Cramer-Rao inequality.

12. (a) State the properties of methods of moments.

(OR)

- (b) Let $X_1, X_2, \dots, X_n$ are a random sample from the uniform distribution with probability density function is $f(x, \theta) = \begin{cases} \frac{1}{\theta} & 0 < x < \theta, \theta > 0 \\ 0 & \text{elsewhere} \end{cases}$. Obtain the maximum likelihood estimator for θ .

13. (a) Elucidate the types of error with examples.

(OR)

- (b) Explain the types of hypothesis with its applications.

14. (a) Obtain the equation of the normal curve that may be fitted to the following data.

Class : 60-65 65-70 70-75 75-80 80-85 85-90 90-95 95-100

Frequency: 3 21 150 335 326 135 26 4

Obtain the expected normal frequencies and test the goodness of fit

(OR)

- (b) What is contingency table? Describe how the chi-square distribution may be used to test whether the two criteria of classification in an $m \times n$ contingency table are independent.

15. (a) Illustrate the procedure of missing plot technique for RBD.

(OR)

- (b) Explain the principles of experimental design.

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fifth Semester)

Branch – MATHEMATICS

REAL ANALYSIS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

- 1 Which of the following is not a property of norm in general?

(i) $ x \geq 0$	(ii) $ x + y \leq x + y $
(iii) $ kx = k x $	(iv) $ x = 0$ iff $x = 0$
- 2 Which of the following is uncountable?

(i) The set of all integers	(ii) The set of rational numbers in $[0,1]$
(iii) The unit interval $[0,1]$	(iv) Every subset of a countable set
- 3 If a set S is compact, then which of the following is true?

(i) S is closed & bounded	(ii) S is closed but not bounded
(iii) S is open & bounded	(iv) S is bounded but not closed
- 4 A metric space in which every Cauchy sequence converges is said to be

(i) bounded sequence	(ii) monotone sequence
(iii) unbounded	(iv) complete
- 5 A mapping f of a set E into R^k is said to be bounded if there is a real number M such that _____ for all $x \in E$.

(i) $f(x) \leq M$	(ii) $ f(x) \leq M$
(iii) $f(x) \geq M$	(iv) $ f(x) \geq M$

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 3 = 15)

- 6 a If a and b are positive real numbers and n is a positive integer, then show that $(ab)^{1/n} = a^{1/n}b^{1/n}$.
 OR
 b Let z and w be complex numbers. Then show that

(i) $ z > 0$ unless $z = 0$, $ 0 = 0$.	
(ii) $ \bar{z} = z $,	
(iii) $ zw = z w $,	
(iv) $ z + w \leq z + w $.	
- 7 a Prove that every infinite subset of a countable set A is countable.
 OR
 b Let $\{E_\alpha\}$ be a (finite or infinite) collection of sets E_α . Then show that $(\cup_\alpha E_\alpha)^c = \cap_\alpha (E_\alpha^c)$.
- 8 a Prove that compact subsets of metric spaces are closed.
 OR
 b If E is an infinite subset of a compact set K , then show that E has a limit point in K .

Cont...

- 9 a If $\{P_n\}$ is a sequence in a compact metric space X , then prove that some subsequence of $\{P_n\}$ converges to a point of X .
OR
b Show that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.
- 10 a Let f be a continuous mapping of a compact metric space X into a metric space Y . Then prove that f is uniformly continuous on X .
OR
b If f is a continuous mapping of a metric space X into a metric space Y , and if E is a connected subset of X , then prove that $f(E)$ is connected.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a Prove that for every real $x > 0$ and every integer $n > 0$ there is one and only real y such that $y^n = x$.
OR
b (i) If $x \in R, y \in R$ and $x > 0$, then show that there is a positive integer n such that $nx > y$.
(ii) If $x \in R, y \in R$ and $x < y$, then show that there is a $p \in Q$ such that $x < p < y$.
- 12 a Let $\{E_n\}, n = 1, 2, 3, \dots$ be a sequence of countable sets and put $S = \bigcup_{n=1}^{\infty} E_n$. Then prove that S is countable.
OR
b If X is a metric space and $E \subset X$, then show that
(i) $\bar{E}$ is closed
(ii) $E = \bar{E}$ if and only if E is closed.
(iii) $\bar{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.
- 13 a Suppose $K \subset Y \subset X$. Then prove that K is compact relative to X if and only if K is compact relative to Y .
OR
b Prove that every k -cell is compact.
- 14 a Suppose $\{s_n\}, \{t_n\}$ are complex sequences, and $\lim_{n \rightarrow \infty} s_n = s, \lim_{n \rightarrow \infty} t_n = t$. Then show that
(i) $\lim_{n \rightarrow \infty} (s_n + t_n) = s + t$;
(ii) $\lim_{n \rightarrow \infty} (cs_n) = cs, \lim_{n \rightarrow \infty} (c + s_n) = c + s$ for any number c ;
(iii) $\lim_{n \rightarrow \infty} (s_n t_n) = st$.
OR
b State and Prove Root Test.
- 15 a Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y .
OR
b Suppose f is a continuous mapping of a compact metric space X into a metric space Y . Then prove that $f(X)$ is compact.

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fifth Semester)

Branch -- MATHEMATICS

LINEAR ALGEBRA

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

1. Which of the following is a symmetric matrix?

(i) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$

(iv) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

2. Let V and W be vector spaces over a field F . A mapping $T: V \rightarrow W$ is called a linear transformation if _____

(i) $T(u + v) = T(u) + T(v)$

(ii) $T(\alpha u) = \alpha T(u)$

(iii) $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$

(iv) $T(\alpha + \beta) = T(\alpha) + T(\beta)$ where $u, v \in V$ and $\alpha, \beta \in F$.

3. Let V be a vector space of polynomials with inner product defined by

$\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. if $f(t) = t$ and $g(t) = t + 1$ then $\langle f, g \rangle =$ _____

(i) $\frac{1}{6}$

(ii) $\frac{5}{6}$

(iii) 0

(iv) $-\frac{5}{6}$

4. The augmented matrix (AB) of a system of linear equation is reduced to the form

$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ then

(i) rank $A = 2$; rank $(AB) = 2$

(ii) rank $A = 2$; rank $(AB) = 3$

(iii) rank $A = 3$; rank $(AB) = 2$

(iv) rank $A = 3$; rank $(AB) = 3$

5. The eigen values of the matrix I_2 are _____

(i) -1, 1

(ii) -1, -1

(iii) 1, -1

(iv) 1, 1

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 3 = 15)

6 a. Find the product of the matrices $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$.

OR

b. Write the all permutation matrices of order 3.

7 a. Prove that $L(S)$ is a subspace of V .

OR

b. If V is the internal direct sum of $U_1, U_2, \dots, U_n$ then Show that V is isomorphic to the external direct sum of $U_1, U_2, \dots, U_n$.

8 a. Prove that $A(A(W)) = W$.

OR

b. Analyze the statement $\dim_F V = m$ then $\dim_F \text{Hom}(V, V) = m^2$.

Cont...

- 9 a. Compute the rank of the matrix $\begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 3 & 1 & 1 & -1 & 2 \\ 4 & 0 & 1 & 0 & 3 \\ 9 & -1 & 2 & 3 & 7 \end{pmatrix}$.

OR

- b. Determine the characteristic roots of the matrix $A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 2 & 0 \\ 7 & -3 & -1 \end{pmatrix}$.

- 10 a. Apply the abstract concept of vector space, prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ iff for some $v \neq 0$ in V , $vT = \lambda v$.

OR

- b. If A is an algebra with unit element, over F , then prove that A is isomorphic to a sub-algebra of $A(V)$ for some vector space V over F .

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

- 11 a. Prove that the matrix $U = \frac{1}{5} \begin{pmatrix} -1 + 2i & -4 - 2i \\ 2 - 4i & -2 - i \end{pmatrix}$ is unitary.

OR

- b. Compute the inverse of $\begin{pmatrix} 2 & 1 & -2 \\ 1 & -1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$.

- 12 a. If $v_1, v_2, \dots, v_n$ is a basis of V over F and if $w_1, w_2, \dots, w_m$ in V are linearly independent over F , then Prove that $m \leq n$.

OR

- b. If V is finite-dimensional and if W is a subspace of V , then Prove that W is finite-dimensional, $\dim W \leq \dim V$ and $\dim v/W = \dim V - \dim W$.

- 13 a. If V is a finite-dimensional inner product space and if W is a subspace of V , then Prove that $V = W + W^\perp$. More particularly, V is the direct sum of W and $W^\perp$.

OR

- b. Analyze the statement If V is finite-dimensional and $v \neq 0 \in V$, then there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

- 14 a. Determine the characteristic vectors and the characteristic subspace corresponding to any one of the characteristic roots of the following matrix $\begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

OR

- b. Verify the Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

- 15 a. If V is finite dimensional over F , then for $S, T \in A(V)$, prove the following

i) $r(ST) \leq r(T)$

ii) $r(ST) \leq \min \{ r(T), r(S) \}$

iii) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

OR

- b. Apply the abstract concept of vector space if $T \in A(V)$ and if $\dim_F V = n$, then Prove that T can have at most n distinct characteristics in F .

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Fifth Semester)

Branch – MATHEMATICS
PYTHON PROGRAMMING

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. Which of the following provides a wide spectrum of mathematical functions and numerical routines for python.
(i) Scipy (ii) Numpy
(iii) Pandas (iv) Matplotlib
2. The List elements are defined by a pair of _____ brackets.
(i) () (ii) [[]]
(iii) [] (iv) { }
3. What is the correct way to plot a line?
(i) plt(x,y) (ii) plt.plot(x,y)
(iii) plot(x,y) (iv) plot.plt(x,y)
4. Which key word is used for function?
(i) fun (ii) define
(iii) def (iv) function
5. The ____ function creates an array of num floats uniformly distributed on the interval from 0 to 1.
(i) rand (num) (ii) rand()
(iii) rand(int) (iv) rand.num()

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a. List three advantages of python programming.
OR
b. Explain the usage of Matplotlib library in python.
- 7 a. Infer String. Explain any three string functions with example.
OR
b. Write a python script reading data from a csv file.
- 8 a. Write a python program to solve quadratic equation $ax^2+bx+c=0$ using if else statement.
OR
b. Outline the for loop with syntax and example.
- 9 a. Summarize the user defined functions with example.
OR
b. Write a python code to plot the sinc(x) using where function.
- 10 a. Discuss random functions with example.
OR
b. Describe how do you read data from the excel file in python.

Cont...

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks

(5 x 6 = 30)

11 a. Explain Ipython recap commands with example.

OR

b. Enumerate the Numpy arrays with example.

12 a. Write a python program for the following (i) Create a list (ii) range function
(iii) slicing the list elements.

OR

b. Paraphrase the input and output functions with example.

13 a. Write a python program to create a wavy pulse.

OR

b. Discuss with example how do you display a mathematical formula in your plot.

14 a. Write python code to implement linear regression algorithm.

OR

b. Explain passing list and arrays to a function with suitable example.

15 a. Explain different types dates and time series functions with example.

OR

b. Illustrate how do you get data from the web with example.

Z-Z-Z

END

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(Second Semester)

Branch - MATHEMATICS

CALCULUS – II

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{10+n}} = ?$ a) 0 b) 1 c) $\sqrt{10}$ d) ∞ .	K1	CO1
	2	Which of the following is false? a) $\sum \frac{1}{n^2}$ is convergent b) $\sum \frac{1}{n^{\frac{3}{2}}}$ is convergent c) $\sum \frac{1}{n}$ is convergent d) $\sum \frac{1}{\sqrt{n}}$ is divergent.	K2	CO1
2	3	The series $\sum \frac{(-1)^n}{n}$ is _____. a) divergent b) absolutely convergent c) not absolutely convergent d) not conditionally convergent.	K1	CO2
	4	Apply Cauchy's root test the series $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ is _____ a) convergent b) divergent c) neither convergent nor divergent d) both convergent and divergent.	K2	CO2
3	5	The radius of convergence of geometric series is _____. a) 0 b) 1 c) r d) ∞ .	K1	CO3
	6	The inequality $ R_n \leq \frac{M}{(n+1)!} x - a ^{n+1}$ is due to _____. a) Laurent b) Maclaurin c) Gregory d) Taylor.	K2	CO3
4	7	If $\vec{f} = xz^3\vec{i} - 2xyz\vec{j} + xz\vec{k}$ then $\text{div } \vec{f}$ at (1, 2, 0) is _____. a) 0 b) 1 c) 2 d) 3.	K1	CO4
	8	A vector $\vec{F}$ is said to be irrotational if _____. a) $\nabla \cdot \vec{F} \neq 0$ b) $\nabla \cdot \vec{F} = 0$ c) $\text{div } \vec{F} = 0$ d) $\nabla \times \vec{F} = 0$.	K2	CO4
5	9	The parametric representation for the cylinder $x^2 + y^2 = 4, 0 \leq z \leq 1, 0 \leq \theta \leq 2\pi$ is _____. a) $x = 2\cos \theta, y = 2\sin \theta, z = z$ b) $x = 2\sin \theta, y = 2\cos \theta, z = z$ c) $x = 4\cos \theta, y = 4\sin \theta, z = 2z$ d) $x = 2\sin \theta, y = 2\cos \theta, z = 2z$.	K1	CO5
	10	Stokes theorem connects _____. a) surface integral and volume integral b) volume integral and surface integral c) line integral and surface integral d) line integral and volume integral.	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that the harmonic series is divergent.	K3	CO1
		(OR)		
	11.b.	Discuss the convergence of $\sum (\sqrt[3]{n^3 + 1} - n)$.		
2	12.a.	Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(a+n)^p(b+n)^q}$ ($a, b, p, q > 0$).	K4	CO2
		(OR)		
	12.b.	Test the convergence of the series $\sum_{n=0}^{\infty} \frac{n^3+1}{2^{n+1}}$.		
3	13.a.	For what values of x do the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converge?	K4	CO3
		(OR)		
	13.b.	Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at $a = 8$.		
4	14.a.	Find f if $\nabla f = 8xy\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$.	K3	CO4
		(OR)		
	14.b.	Using Green's theorem evaluate $\int_C x^2 y dx + x^2 dy$ where C is the triangle joining the points $(0, 0)$, $(1, 0)$ and $(0, 1)$.		
5	15.a.	Evaluate $\iint \vec{A} \cdot \vec{n} dS$ if $\vec{A} = yz\vec{i} + 2y^2\vec{j} + xz^2\vec{k}$ and S is the surface $x^2 + y^2 = 9$ contained in the first octant between the planes $z = 0$ and $z = 2$.	K5	CO5
		(OR)		
	15.b.	Using Gauss divergence theorem evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Examine the convergence $\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+1} \right)^{1/2}$.	K4	CO1
2	17	Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{3n^3+7n^2}{5n^9+11} \right) x^n$.	K4	CO2
3	18	Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.	K4	CO3
4	19	Prove that $\text{div} (r^n \vec{r}) = (n+3)r^n$.	K5	CO4
5	20	Verify Stoke's theorem $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ and C is the circle $x^2 + y^2 = 1$ boundary of the unit sphere.	K5	CO5

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2024
(First Semester)

Branch - MATHEMATICS

ANALYTICAL GEOMETRY OF 3D AND TRIGONOMETRY

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The equation of the x-axis is _____. (a) $y = 0, z = 0$ (b) $x = 0, z = 0$ (c) $x = 0, y = 0$ (d) none	K1	CO1
	2	If the two lines are coplanar, they must _____. (a) equal (b) intersect (c) parallel (d) none	K2	CO1
2	3	When the radius of the radius $u^2 + v^2 + w^2 - d$ is negative, the locus is an _____. (a) real sphere (b) point sphere (c) Imaginary sphere (d) greatest sphere	K1	CO2
	4	Intersection of two spheres is a _____. (a) sphere (b) great circle (c) real sphere (d) circle	K2	CO2
3	5	A _____ is a surface generated by a line which passes through a fixed point and makes a constant angle with the fixed line through the fixed point. (a) right circular cone (b) circle (c) quadric cone (d) reciprocal cone	K1	CO3
	6	The cone touching the coordinate planes is _____ to the cone passing through the coordinates axes. (a) equal (b) reciprocal (c) opposite (d) not equal	K2	CO3
4	7	Any plane which intersects the cylinder whose equation is of second degree is a _____. (a) right circular cylinder (b) enveloping cylinder (c) conicoid (d) conic	K1	CO4
	8	The equation of the tangent plane is _____. (a) $axx_1 + byy_1 + czz_1 = 1$ (b) $axx_1 + byy_1 + czz_1 = 0$ (c) $alx_1 + bmy_1 + cnz_1 = 1$ (d) $alx_1 + bmy_1 + cnz_1 = 0$	K2	CO4
5	9	$x_n + \frac{1}{x_n} =$ _____. (a) $\cos n\theta$ (b) $\sin n\theta$ (c) $2 \cos n\theta$ (d) $2i \sin n\theta$	K1	CO5
	10	$2 \cosh^2 x - 1 =$ _____. (a) $\cosh nx$ (b) $\cosh 2x$ (b) $\sinh 2x$ (d) $\cosh^2 2x$	K2	CO5

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the symmetrical form of the equations of the line of intersection of the planes $x + 5y - z - 7 = 0$, $2x - 5y + 3z + 1 = 0$.	K2	CO1
		(OR)		

Cont...

1	11.b.	Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plane through them.	K2	CO1
2	12.a.	Find the equation to the sphere through the four points (2,3,1), (5,-1,2), (4,3,-1) and (2,5,3).	K2	CO2
	12.b.	(OR) Find the equation of a sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point (2,-2,1) and passes through the origin.		
3	13.a.	Find the equation of the cone with vertex O and base curve, the conic in which the surface $ax^2 + by^2 + cz^2 = 1$ is cut by the plane $l_1x + m_1y + n_1z = p$.	K2	CO3
	13.b.	(OR) Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{27}$.		
4	14.a.	Find the equation of the cylinder whose generators are parallel to the z axis and guiding curve is $ax^2 + by^2 = cz, lx + my + nz = p$.	K3	CO4
	14.b.	(OR) Find the tangents and tangent planes to the conicoid.		
5	15.a.	Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of $\cos \theta$.	K3	CO5
	15.b.	(OR) If $\cos(x + iy) = \cos \theta + i \sin \theta$, prove that $\cos 2x + \cosh 2y = 2$.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	The straight lines $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$; $\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1}$ are cut by a third whose direction cosines are λ, μ, ν . Show that the length intercepted on the third line is given by $\left \frac{\alpha - \alpha_1}{l} - \frac{\beta - \beta_1}{m} - \frac{\gamma - \gamma_1}{n} \right + \left \frac{l}{l_1} - \frac{m}{m_1} - \frac{n}{n_1} \right $ and deduce the length of the shortest distance.	K2	CO1
2	17	Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$, $x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.	K3	CO2
3	18	Find the angle between the lines in which the plane $ux + vy + wz = 0$ cuts the cone.	K3	CO3
4	19	Find the equation of the right circular cylinder described on the circle through the points (a,0,0), (0,a,0), (0,0,a) as a guiding curve.	K4	CO4
5	20	Expand $\sin^3 \theta \cos^5 \theta$ in a series of sines of multiples of θ .	K3	CO5