

**PSG COLLEGE OF ARTS & SCIENCE**  
(AUTONOMOUS)  
**BSc DEGREE EXAMINATION MAY 2024**  
(First Semester)  
Branch – **MATHEMATICS**  
**CALCULUS - I**

Time: Three Hours

Maximum: 75 Marks

**SECTION-A (10 Marks)**

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The curve is called twisted cubic when the vector equation $r(t)$ is _____. a. $\langle t, t^2 \rangle$ b. $\langle t, t^2, t^3 \rangle$ c. $\langle t, t^2, t^{-1} \rangle$ d. $\langle t, t^{-1} \rangle$	K1	CO1
	2	A semicubical parabola curve is made up of a finite number of smooth curve is called _____. a. smooth curve                b. piecewise smooth c. cusp                              d. curvature	K2	CO1
2	3	The range of $g(x,y)=\sqrt{9-x^2-y^2}$ is a. (0,3)                      b. [1,3] c. [0,3]                      d. (1,3)	K1	CO2
	4	Laplace equation is _____. a. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ b. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$ c. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ d. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u_x$	K2	CO2
3	5	The gradient of $f(x,y,z)=x \sin yz$ is _____. a. $\langle \sin y, \sin z, \sin xz \rangle$ b. $\langle \sin yz, xz \cos yz, xy \cos yz \rangle$ c. $\langle \sin z, xz \cos z, xy \cos y \rangle$ d. $\langle \sin y, xy \cos y, xz \cos z \rangle$	K1	CO3
	6	A point (a, b) is called critical point if _____. a. $f_x(a,b)=0$ b. $f_y(a,b)=0$ c. $f_x(a,b)=0$ & $f_y(a,b)=0$ d. all the above	K2	CO3
4	7	The moment of inertia of a particle of mass m and r distance of particle to the x axis is _____. a. mr                              b. m/r c. $mr^2$ d. $m/r^2$	K1	CO4
	8	A plane region lies in between the graphs of two continuous functions of x is called _____. a. type I                              b. type II c. integrable                        d. double integrable	K2	CO4
5	9	The length of the longest diagonal of all boxes of partition P is called _____. a. modulus                        b. norm c. center of mass                d. moments of inertia	K1	CO5
	10	Jacobian of the transformation T given by $x=g(u,v)$ and $y=h(u,v)$ is _____. a. $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$ b. $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ c. $\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$ d. $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix}$	K2	CO5

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**SECTION - B (35 Marks)**

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that if $ r(t) =c$ then $r'(t)$ is orthogonal to $r(t)$ for all $t$ .	K2	CO1
		(OR)		
	11.b.	Show that the curvature of a circle of radius $a$ is $1/a$ .		
2	12.a.	Sketch the level curve of the function $f(x,y)=6-3x-2y$ for the value $k=-6, 0, 6, 12$ .	K2	CO2
		(OR)		
	12.b.	Find the second partial derivatives of $f(x,y)=x^3+x^2y^3-2y^2$		
3	13.a.	If $z=x^2y+3xy^4$ where $x=e^t$ and $y=\sin t$ , find $\frac{dz}{dt}$ .	K3	CO3
		(OR)		
	13.b.	Find the extreme values of $f(x,y)=x^2+2y^2$ on the disk.		
4	14.a.	Evaluate the iterated integral $\int_0^3 \int_1^2 x^2 y dy dx$	CO4	K4
		(OR)		
	14.b.	Find the volume of the solid bounded by the plane $z=0$ and the paraboloid $z=1-x^2-y^2$		
5	15.a.	Evaluate $\iiint_B xyz^2 dV$ where $B$ is the rectangular box given by $B=\{(x,y,z)/0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$	CO4	K5
		(OR)		
	15.b.	Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$		

**SECTION - C (30 Marks)**

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find the curvature of the twisted cubic $r(t)=\langle t, t^2, t^3 \rangle$ at a general point and at $(0,0,0)$	K1	CO2
2	17	Show that the function $u(x,y)=e^x \sin y$ is a solution of Laplace's equation. Also find the tangent plane to the elliptic paraboloid $z=2x^2+y^2$ at the point $(1,1,3)$ .	K2	CO2
3	18	Find the shortest distance from the point $(1, 0, -2)$ to the plane $x+2y+z=4$ .	K3	CO3
4	19	Find the mass and center of mass of a triangular lamina with vertices $(0,0)$ , $(1,0)$ and $(0,2)$ if the density function is $p(x,y)=1+3x+y$ .	K4	CO4
5	20	Find the center of mass of a solid of constant density that is bounded by the parabola cylinder $x=y^2$ and the planes $x=z$ , $z=0$ and $x=1$ .	K4	CO4