

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)

Branch – MATHEMATICS

COMPLEX ANALYSIS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

1. A function which is analytic at every point of the complex plane is ---- function
(i) continuous (ii) entire
(iii) bounded (iv) differentiable
2. Invariant points of $w = \frac{1+z}{1-z}$ are
(i) ± 1 (ii) $\pm i$
(iii) ± 2 (iv) 0,0
3. $\int_C \frac{dz}{z-3} = \text{----}$ where C is the circle $|z - 2| = 5$
(i) $2\pi i$ (ii) $4\pi i$
(iii) $6\pi i$ (iv) πi
4. The singularity of the function $\frac{\sin z}{z}$ at $z = 0$ is
(i) essential singularity (ii) simple pole
(iii) removable singularity (iv) double pole
5. Residue at $z = 2$ of the function $f(z) = \frac{2z+1}{z^2-z-2}$ is
(i) $\frac{5}{3}$ (ii) $\frac{1}{3}$
(iii) $\frac{3}{5}$ (iv) $\frac{2}{3}$

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

6. (a) Verify C-R equations for the function $f(z) = z^3$.
(OR)
(b) Derive the complex form of C-R equations.
7. (a) Prove that any bilinear transformation transform real axis into itself.
(OR)
(b) Find the bilinear transformation which sends the points $-1, 1, \infty$ into $-i, -1, i$ respectively.
8. (a) Evaluate $\int_C f(z)dz$ where $f(z) = y - x - i3x^2$ and C is the line segment from
 $z = 0$ to $z = 1 + i$

(OR)

Cont...

(b) State Cauchy's integral formula and find $\int_C \frac{dz}{z-2}$ where C is the circle
 $|z-2|=5$.

9. (a) Expand $\cos z$ into a Taylor's series about the point $z = \frac{\pi}{2}$ and determine the region of convergence.

(OR)

(b) State and Prove Riemann's Theorem.

10. (a) Evaluate the residue of $\frac{z+1}{z^2-2z}$ at its poles.

(OR)

(b) Calculate the residue at $z=0$ of $\cot z$ at $z=0$.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

11.(a) Verify whether $f(z) = e^x(\cos y - i \sin y)$ is nowhere differentiable or not.
 (OR)

(b) Find the analytic function $f(z) = u + iv$ if $u + v = \frac{\sin 2x}{\cos 2y - \cos 2x}$.

12. (a) Any bilinear transformation preserves cross ratio. Justify this statement.
 (OR)

(b) Prove that any bilinear transformation which maps the unit circle $|z|=1$ onto the unit

circle $|w|=1$ can be written in the form $w = e^{i\lambda} \left(\frac{z-\alpha}{\bar{\alpha}z-1} \right)$ where λ is real.

13. (a) Prove that $\left| \int_a^b f(t) dt \right| \leq \int_a^b |f(t)| dt$.
 (OR)

(b) State and prove Cauchy's theorem.

14. (a) State and prove Taylor's Theorem.
 (OR)

(b) Find the Laurent's series expansion of the function $\frac{z+4}{(z+3)(z-1)^2}$ in
 (i) $0 < |z-1| < 4$ and (ii) $|z-1| > 4$.

15. (a) State and prove Cauchy's Residue Theorem.
 (OR)

(b) Evaluate $\int_0^\infty \frac{dx}{1+x^4}$ using contour integration method.

Z-Z-Z END