20MAU22

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024
(Sixth Semester)
Branch – **MATHEMATICS**

GRAPH THEORY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 A graph G is said to be _____ graph if there is atleast one path between every pair of vertices in G,
(i) Disconnected (ii) connected
(iii) Components (iv) tree

2 Every non trivial tree has atleast two vertices of degree_____
(i) 1 (ii) 0
(iii) Same (iv) 2

3 K_5 is a_____ graph
(i) Planar (ii) non-planar
(iii) Non-complete (iv) Tree

4 The reduced incidence matrix of a tree is ____
(i) Singular (ii) non-circuit
(iii) Non-singular (iv) isomorphic

5 ___is a vertex in which the in degree and the out degree are both equal to zero .
(i) Pendent vertex (ii) complete vertex
(iii) Null vertex (iv) isolated vertex

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Show that the number of vertices of odd degree in a graph is always even.
OR
b Explain operations on graphs.

7 a Prove that a graph G with n vertices, n-1 edges and no circuits is connected.
OR
b Every connected graph has at least one spanning tree. Justify this statement.

8 a Prove that Kuratowski's second graph is also non-planar.
OR
b Provide an example of a graph which is both Eulerian and planar.

9 a If A(G) is an incidence matrix of a connected graph G with n vertices then prove that the rank of A(G) is n-1.
OR
b Explain any three observations of path matrix

10 a (i) Examine equivalence relations with examples.
OR
b Discuss about three observations on the properties of adjacency matrix of a digraph .

Cont...

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a A connected graph G is an Euler graph⇔iff all vertices of G are of even degree. Analyse this statement.

OR

b Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

12 a (i) Prove that every tree has either one or two centers.
(ii) Prove that there is one and only path between every pair of vertices in a tree T.

OR

b Prove that a tree with n vertices has n-1 edges.

13 a State and prove Euler's formula for a connected planar graph.

OR

b Prove that the complete graph of five vertices is non-planar.

14 a If B is a circuit matrix of a connected graph G with e edges and n vertices, then prove that Rank of B= e-n+1.

OR

b If A is circuit matrix and B is incidence matrix, prove that $A.B^T=B.A^T=0$. (where T denotes transposed matrix).

15 a Describe the types of Digraph with illustrations.

OR

b Prove that the determinant of every square sub matrix of A, the incidence matrix of a digraph is 1,-1 or 0.

Z-Z-Z END