

PSG COLLEGE OF ARTS & SCIENCE  
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2024  
(Third Semester)

Branch – MATHEMATICS

**PARTIAL DIFFERENTIAL EQUATIONS & FOURIER TRANSFORMS**

Time: Three Hours

Maximum: 50 Marks

**SECTION-A (5 Marks)**

Answer ALL questions

ALL questions carry EQUAL marks (5 x 1 = 5)

- 1 The partial differential equation by eliminating the constants from  $z = ax + by + ab$  is \_\_\_\_\_.  
 (i)  $z = px + qy + pq$  (ii)  $z = px + qy - pq$   
 (iii)  $z = px + qy$  (iv)  $z = px - qy$
- 2 If  $u_{xx} + x^2 u_{yy} = 0$  then the given PDE equation is \_\_\_\_\_.  
 (i) Parabolic (ii) Elliptic  
 (iii) Hyperbolic (iv) Mixed type
- 3 If  $f(x)$  is an even function then \_\_\_\_\_.  
 (i)  $f(-x) = -f(x)$  (ii)  $f(-x) = f(x)$   
 (iii)  $f(-x) = -1/f(x)$  (iv) 0
- 4 The Fourier cosine transform of  $e^{-x^2/2} =$  \_\_\_\_\_.  
 (i)  $e^{-w^2/2}$  (ii)  $-e^{-w^2/2}$   
 (iii)  $e^{w^2/2}$  (iv)  $-e^{w^2/2}$
- 5 If  $\partial u / \partial t = 0$  then the heat equation becomes \_\_\_\_\_ equation.  
 (i) Diffusion (ii) Laplace  
 (iii) Wave (iv) Poisson

**SECTION - B (15 Marks)**

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 3 = 15)

- 6 a Form the partial differential equation by eliminating the arbitrary function from  $f(x + y + z, x^2 + y^2 + z^2) = 0$ .  
 OR  
 b Find the complete integral of  $p^2 + q^2 = x + y$ .
- 7 a Find the characteristics of the equation  $u_{xx} + 2u_{xy} + \sin^2(x)u_{yy}u_y = 0$ .  
 OR  
 b Solve  $(D^2 + 5D + 6)u = e^x$ .
- 8 a Find the Fourier coefficient  $a_n$  of the periodic function  $f(x) = |x|$  for  $-\pi < x < \pi$ .  
 OR  
 b Find the Fourier series of the function  

$$f(x) = \begin{cases} -k & \text{if } -2 < x < 0 \\ k & \text{if } 0 < x < 2 \end{cases} \quad \text{and } L = 2.$$

Cont...

- 9 a Consider the periodic rectangular wave  $f_L(x)$  of period  $2L > 2$  given by

$$f(x) = \begin{cases} 0 & \text{if } -L < x < -1 \\ 1 & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < L \end{cases}$$

OR

- b Find the Fourier transform of  $xe^{-x^2}$ .
- 10 a Find the temperature  $u(x, t)$  in a laterally insulated copper bar 80 cm long if the initial temperature is  $100\sin(\pi x/80)^\circ\text{C}$  and the ends are kept at  $0^\circ\text{C}$ . How long will take for the maximum temperature in the bar to drop to  $50^\circ\text{C}$ ? Calculate the physical data for copper: density  $8.92 \text{ g/cm}^3$ , specific heat  $0.092 \text{ cal/(g}^\circ\text{C)}$ , thermal conductivity  $0.95 \text{ cal/(cm sec}^\circ\text{C)}$ .

OR

- b Solve the heat equation  
 $\frac{\partial u}{\partial t} = c^2 \partial^2 u / \partial x^2$ ,  $u(x, 0) = f(x)$ ,  $(-\infty < x < \infty)$ , where  $f(x)$  is the given initial temperature of the bar.

**SECTION -C (30 Marks)**

Answer ALL questions

ALL questions carry EQUAL Marks (5 x 6 = 30)

- 11 a Find the general integral of the linear PDE  $pz - qz = z^2 + (x + y)^2$ .  
 OR  
 b Find the complete integral of the PDE  $z^2 = pqxy$ .
- 12 a Reduce the equation  $(1 + x^2)u_{xx} + (1 + y^2)u_{yy} + xu_x + yu_y = 0$  to a canonical form.  
 OR  
 b Solve  $(D^2 + 16)u = e^{-3x} + \cos 4x$ .
- 13 a Find the Fourier series of the given function  $f(x) = x^2$ ,  $(-\pi < x < \pi)$  which is assumed to have the period  $2\pi$ .  
 OR  
 b Find the Fourier series of the given function  $f(x) = x + \pi$ ,  $(-\pi < x < \pi)$  and  $f(x + 2\pi) = f(x)$ .
- 14 a Find the Fourier cosine transform of  $\mathcal{F}_c(e^{-ax})$  of  $f(x) = e^{-ax}$ , where  $a > 0$ .  
 b Find the Fourier transform of  $f(x) = 1$  if  $|x| < 1$  and  $f(x) = 0$  otherwise.
- 15 a Find the temperature in a laterally insulated bar of length  $L$  assuming that whose ends are insulated,

$$f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$$

OR

- b Solve the temperature in the infinite bar if the initial temperature is given below by the method of convolution.

$$f(x) = \begin{cases} U_0 = \text{const} & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$