# PSG COLLEGE OF ARTS & SCIENCE
## (AUTONOMOUS)

### BSc DEGREE EXAMINATION MAY 2024
(Third Semester)

Branch – **MATHEMATICS**

**MATHEMATICAL STATISTICS - I**

Time: Three Hours                                                    Maximum: 50 Marks

## SECTION-A (5 Marks)
### Answer ALL questions
**ALL** questions carry **EQUAL** marks                  (5 x 1 = 5)

1   The numerical evaluation of a chance factor of an experiment is called
   (i) Random variable                 (ii) Event
   (iii) Probability                       (iv) Axioms

2   A variable whose value is determined by an outcome of a random experiment is called
   (i) Continuous random variable      (ii) Discrete random variable
   (iii) Random variable                 (iv) Pseudo variable

3   The formula for moment generating function in continuous random variable

   (i) $E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} P(x)$          (ii) $E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} f(x)$

   (iii) $E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx} f(y)$         (iv) $E(e^{tx}) = \sum_{x=0}^{n} e^{tx} P(x)$

4   The shape of the normal distribution is
   (i) Symmetric       (ii) Circle       (iii) Curve        (iv) Bell

5.  The number of normal equations involved in fitting a straight line trend.
   (i) 1                  (ii)2               (iii) 3               (iv) 4

## SECTION - B (15 Marks)
### Answer ALL Questions
**ALL** Questions Carry **EQUAL** Marks                  (5 x 3 = 15)

6   a   For a fixed B with P(B)>0 , then prove that P(A/B) is a probability function.
   **OR**
   b   For any three events A,B, and C then prove
   that $P\left[\dfrac{A \cup B}{C}\right] = P\left[\dfrac{A}{C}\right] + P\left[\dfrac{B}{C}\right] - P\left[\dfrac{A \cap B}{C}\right]$

7   a   A continuous random variable X has the probability function
   $f(x) = k(1+x), \ 2 \le x \le 5$. Find $P(X < 4)$
   **OR**
   b   A random variable X has the following probability distribution.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|-----|----|----|---|---|---|---|
| $p(x)$ | 0.1 | k | 0.2 | 2k | 0.3 | k |

   Find (i) The value of 'k' (ii) Evaluate P(X<2) and P(-2<x<2) (iii) Find the cumulative distribution of X and (iv) Evaluate the mean of X.

8   a   The p.m.f of a RV X, is given by $p(X = j) = \dfrac{1}{2^j}, \ j = 1,2,3...$  Find MGF, mean and variance.
   **OR**
   b   If the joint pdf of (X,Y) is given by $f(x,y) = 2 - x - y$ in $0 \le x \le y \le 1$, find E(X)

**Cont...**

9 a The mean and variance of the binomial distribution are 4 and 4/3 respectively. Find $P(X \geq 1)$

**OR**

b In a Poisson frequency distribution, frequency corresponding to 3 successes is 2/3 times frequency corresponding to 4 successes. Find the mean and standard deviation of the distribution.

10 a Explain the lottery method of simple random sampling.

**OR**

b Fit a straight line for the following data.

| X | 2 | 3 | 5 | 8 | 10 |
|---|---|---|---|---|----|
| Y | 5 | 6 | 10 | 18 | 21 |

## SECTION -C (30 Marks)
### Answer ALL questions
**ALL questions carry EQUAL Marks**      (5 x 6 = 30)

11 a State and prove Bayes's theorem.

**OR**

b The odds against manager X setting the wage dispute with the workers are 8:6 and odds in favour of manager Y setting the same dispute are 14:16 (i) what is the chance that neither settles the dispute, if they both try , independently of each other? (ii) what is the probability that the dispute will be settled?

12 a Find the moment generating function of the random variable 'X' having p.d.f

$$f(x) = \begin{cases} \dfrac{1}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

**OR**

b Let X be a random variable with the following probability distribution

$x$:      -3    6    9

$P(x=x)$:   1/6   1/2   1/3

Find $E(X)$ and $E(X^2)$ and using the laws of expectations , evaluate $E(2X+1)^2$

13 a The joint probability mass function of X, Y is given by

$P(X,Y) = K(2x+3y), \ x = 0,1,2, \ . \ y = 1, 2, 3$ Find all the marginal and conditional distributions.

**OR**

b If the joint probability density function of two dimensional random variable X, Y is given    by $f(x, y) = \begin{cases} K(6-x-y), & 0 < x < 2, \ 2 < y < 4 \\ 0 & \text{other wise} \end{cases}$

Find    (i)    The value of K   (ii) P $(x<1, y<3)$ (iii) P $(x<1/y<3)$

14 a Derive the mean and variance of the density function

$$f(x) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, & -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

**OR**

b Show that for t- distribution with n d.f mean deviation about mean is given

$$\frac{\sqrt{n}\ \Gamma[(n-1)/2]}{\sqrt{\pi}\Gamma(n/2)}$$

15 a Explain the stratified and systematic sampling procedure.

**OR**

b Fit an exponential curve of the form $Y = ab^x$ to the following data

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Y | 1.0 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 |