20MAU01

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2021
(First Semester)
Branch – **MATHEMATICS**

CALCULUS - I

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. A parametrization $r(t)$ is called ________ on the interval I if r' is continuous and $r'(t) \neq 0$ on I.
   i) smooth ii) regular iii) normal iv) constant

2. A function of two variables is called ________ at (a, b)if $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$.
   i) analytic ii) differentiable iii) continuous iv) concentric

3. Two surfaces are called ________ at a point of intersection if their normal lines are perpendicular at that point.
   i) coincide ii) orthonormal iii) orthogonal iv) constant

4. If $m \leq f(x, y) \leq M$ for all (x, y) in D, then $mA(D) \leq \int\int_D f(x, y)\, dA$________.
   i) $< MA(D)$ ii) $< mA(D)$ iii) $\leq mA(D)$ iv) $\leq MA(D)$

5. The triple riemann sum is ________.
   i) $\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ij}^*, y_{ij}^*, z_{ij}^*)\,\Delta V$ ii) $\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ij}, y_{ij}, z_{ij})\,\Delta V$
   iii) $\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ij}^*, y_{ij}^*, z_{ij}^*)\,\nabla V$ iv) $\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ij}, y_{ij}, z_{ij})\,\nabla V$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Describe the curve defined by the vector function $r(t) = \langle 1 + t, 2 + 5t, -1 + 6t\rangle$

OR

b) If $r(t) = \langle f(t), g(t), h(t)\rangle = f(t)i + g(t)j + h(t)k$ where f, g *and* h are differentiable functions then prove that $r'(t) = \langle f'(t), g'(t), h'(t)\rangle = f'(t)i + g'(t)j + h'(t)k$.

7. a) Find the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$.

OR

b) Evaluate $\lim_{(x,y)\to(1,z)}(x^2y^3 - x^3y^2 + 3x + 2y)$.

8. a) If $z = e^x \sin y$, where $x = st^2$ and $y = s^2t$, find $\frac{dz}{ds}$ and $\frac{dz}{dt}$.

OR

b) Find the extreme values of $f(x, y) = y^2 - x^2$.

9. a) Use midpoint rule with m=n=2 to estimate the value of the integral $\int\int_R (x - 3y^2)\, dA$, where $R = \{(x, y)|0 \leq x \leq 2, 1 \leq y \leq 2\}$.

OR

b) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes x=2 and y=2, and the three coordinate planes.

10. a) Evaluate $\int\int\int_B e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, dV$, where B is the unit ball: $B = \{(x, y, z)|x^2 + y^2 + z^2 \leq 1\}$.

OR

b) Find cylindrical coordinates of the point with rectangular coordinates (3,-3,-7).

Cont...

SECTION -C (30 Marks)

Answer ALL questions
ALL questions carry EQUAL Marks (5 x 6 = 30)

11. a) Find the curvature of the twisted cubic $r(t) = \langle t, t^2, t^3 \rangle$ at a general point and at (0,0,0).

OR

b) Find the unit normal and binomial vectors for the circular helix $r(t) = cost\, i + sint\, j + tk$.

12. a) Find the second partial derivative of $f(x,y) = x^3 + x^2y^3 - 2y^2$.

OR

b) Show that $f(x,y) = xe^{xy}$ is differentiable at (1,0) and find its linearization there. Then use it to approximate $f(1.1, -0.1)$.

13. a) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box.

OR

b) Find the equations of the tangent plane and normal line at the point (-2,1,-3) to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

14. a) Evaluate $\int\int_D xy\, dA$, where D is the region bounded the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

OR

b) Find the volume of the solid bounded by the plane z=0 and the paraboloid $z = 1 - x^2 - y^2$.

15. a) Evaluate $\int\int\int_E z\, dV$, where E is the solid tetrahedron bounded by the four planes x=0, y=0, z=0, and x+y+z=1.

OR

b) Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2) dzdydx$.

Z-Z-Z END