PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc(SS) DEGREE EXAMINATION DECEMBER 2023
(Fourth Semester)

Branch – **SOFTWARE SYSTEMS (five year integrated)**

TRANSFORMATION TECHNIQUES

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 What is the Laplace transform of 1?
(i) $\frac{1}{s}$ (ii) s
(iii) $\frac{1}{s^2}$ (iv) s^2

2 Find the x[2] for x[n + 1] – x[n] = n; x[0] =1.
(i) 1 (ii) 2
(iii) 3 (iv) 0

3 What is $f[k]$ if $F(z) = 1 + z^{-1} + z^{-2} + z^{-3} + \dots$?
(i) {0} (ii) 1, 2, 3, …
(iii) –1, –2, –3,… (iv) 1, 1, 1,…

4 Find the Fourier transform of e^{-jta}
(i) $2\pi\delta(\omega + a)$ (ii) $2\pi\delta(\omega - a)$
(iii) $\pi\delta(\omega + a)$ (iv) $\pi\delta(\omega - a)$

5 When one can say d.f.t, F[k] is periodic with period N?
(i) F[k + N] = F[kN] (ii) F[k + N] = F[k]
(iii) F[k + N] = F[N] (iv) F[k + N] = F[1]

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a State the final value theorem and verify the final value theorem for $f(t) = e^{-2t}$.

OR

b Show that $f * g = g * f$ where $f(t) = 2t$ and $g(t) = t^3$.

7 a Show that $x[n] = A2^n$, where A is a constant, is a solution of $x[n+1] - 2x[n] = 0$, x[0]=3.

OR

b Determine x[4] if $2x[k + 2] - x[k + 1] + x[k] = -k^2$, x[0]= 1 and x[1]=3.

8 a Consider the function $f(t) = u(t)e^{-t}$ which has Laplace transform $F(s) = \frac{1}{s+1}$. Suppose $f(t)$ is sampled at intervals T to give the sequence, $f[k] = e^{-kT}$, for k=0, 1, 2,…. Show that provided sample interval T is sufficiently small, $TF^*(s)$ approximates the Laplace transform F(s).

OR

b Show that the z transform of $e^{-k} + k$ is $\frac{z}{z - e^{-1}} + \frac{z}{(z-1)^2}$.

9 a Apply linear property to find the Fourier transform of $(u(t)e^{-t} + u(t)e^{-2t})$.

OR

b Evaluate the Fourier transform for $e^{-jt} f(t) = \begin{cases} 3e^{-jt} & -2 \le t \le 2 \\ 0 & otherwise \end{cases}$.

10 a Determine the inverse d.f.t. of the sequence F[k], for k = 0, 1, 2, 3, given by F[k] = –4, 1, 0, 1.

OR

b Show that the function $\widetilde{F}(\omega) = T\sum_{n=0}^{N-1} f[n]e^{-j\omega nT}$ is periodic with period $\frac{2\pi}{T}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Solve $y'' - y = -t^2$, $y(0) = 2$, $y'(0) = 0$ using the Laplace transform.

OR

b Solve $y'' + y' - 2y = -2$, $y(0) = 2$, $y'(0) = 1$.

12 a A signal is received by a computer in a sampled form from a transducer measuring the height of acetic acid in a large chemical tank. The measurements are known to fluctuate as a result of the acid swilling about in the tank. It is therefore decided to smooth out these fluctuations by averaging the five most recently measured values of the level and to use this moving average as a measure of the height of the acid in the tank. Formulate a difference equation to carry out this averaging and draw a block diagram of the difference equation.

OR

b A computer is fed a signal representing the velocity of an object as a function of time. Prior to entering the computer the signal is sampled using an analogue-to-digital converter. Derive a difference equation and associated block diagram to obtain the acceleration of the object as a function of time.

13 a The continuous signal $f(t) = \cos\left(\frac{\pi t}{2}\right)$ is sampled at 1 second intervals starting from t = 0. Find the Laplace transform of the sampled signal. Has it poles? Justify your answer.

OR

b Solve the difference equation y[k+1] – 3y[k] = 0, y[0] = 4.

14 a Given that when $f(t) = \begin{cases} 1 & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$, $F(\omega) = \frac{2\sin\omega}{\omega}$, apply the second shift theorem to find the Fourier transform of $g(t) = \begin{cases} 1 & 1 \le t \le 3 \\ 0 & otherwise \end{cases}$. Verify the result directly.

OR

b Find the Laplace transform of (i) $u(t)\, e^{-2t}$ (ii) $u(t)e^{2t}$.
Let $s = j\omega$ and analyse the result.

15 a Suppose f[n] is the sequence 3, 9, 2, –1 and g[n] is the sequence –4, 8, 5.
(i) Find the linear convolution h[n]=f*g.
(ii) Develop a graphical interpretation of this process.

OR

b Suppose f[n] = 7, 2, –3 and g[n] = 1, 9, –1. Assume both sequences f and g start at n = 0.
(i) Find the linear cross-correlation c[n] = f*g
(ii) Develop a graphical interpretation of this process.

Z-Z-Z END