22STP313

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2023
(Third Semester)

Branch – **STATISTICS**

STOCHASTIC PROCESSES

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 If $\{X(t), t \in T\}$ is a stochastic process in which X(t) represents the outcome in the t-th throw of a die, the stochastic process is called
(i) Discrete Random Sequence (ii)Discrete random Process
(iii) Continuous Random Sequence (iv) Continuous random Process

2 A state i is a periodic if
(i) $p_{ii}^{(n)} = 0 \; for\; all\; n$ (ii) $p_{ii}^{(n)} > 0 \; for\; all\; n$
(iii) $p_{ii}^{(n)} < 0 \; for\; all\; n$ (iv) $p_{ii}^{(n)} = 0 \; for\; all\; even\; values\; of\; n$

3 Which of the following is not true? The Chapman Kolmogorov equation is used to find-----
(i) Higher order transition probabilities when the TPM is known
(ii) TPM when higher order probabilities are known
(iii) whether a state is periodic
(iv) All the above

4 If the number of arrivals follow Poisson Process, the interarrival time is ________.
(i) Poisson Process (ii) Exponential process
(iii) Gaussian Process (iv) Binomial Process

5 If in a stochastic process $E(X(t)) is\; a\; constant\; and\; E[X(t1)X(t2)]$ depends only on the time difference, the stochastic process is said to be
(i) Wide sense stationary (ii)Weakly Stationary
(iii) Covariance stationary (iv) All the above

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Consider a Markov Chain $\{X_n, n = 1,2,3 - - -\}$ with state space $S = \{0,1,2\}\; and\; the\; initial\; distribution\; (0.3,0.2,0.5)$, find

$$P(X_1 = 0, X_2 = 1, X_3 = 0)\; when\; the\; TPM\; is\; P = \begin{pmatrix} 0.1 & 0.7 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$$

OR

b One of the three children A, B and C throw a ball to any of the other two children. The child A always throws to child B. The chance that the child B will throw the ball to C is 1/3. The chance that C will throw the bal to A is ¼. The three children A, B and C have the chances 2/5, 1/5 and 2/5 respectively to start the game. Compute

$P(X_1 = A, X_2 = B, X_3 = C, X_4 = A)$

Cont...

7 a Find the stationary distribution of the markov chain with TPM.
$$P = \begin{pmatrix} 1/3 & 2/3 \\ 4/5 & 1/5 \end{pmatrix}$$

OR

b State and prove Chapman Kolmogorov equation.

8 a Explain Random walk with an example.

OR

b Derive the Kolmogorov backward equation.

9 a Prove that Poisson process has additive property.

OR

b Prove that the difference between two Poisson processes is not a Poisson process.

10 a Verify whether the Stochastic process $\{X(t), t \in T\}$, is covariance stationary where $X(t) = A \cos(bt+\theta)$. Here A and b are constants and $\theta \sim U(0,2\pi)$

OR

b. Describe the renewal process.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Derive the condition for the state of a Markov chain to be recurrent or transient.

OR

b Verify whether the Markov chain is irreducible if the TPM
$$\text{is } P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

12 a Verify whether the state 1 is null recurrent if the TPM is
$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

OR

b How will you classify the states of a markov chain.

13 a Explain absorption probabilities.

OR

b Explain Random walk theory.

14 a Derive the differential difference equation of pure birth process.

OR

b Derive the mean and variance of Poisson process.

15 a Derive the relation between P(s) and F(s).

OR

b Derive the Renewal equation.

Z-Z-Z END