**22STP101N/ 22STP101**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**MSc DEGREE EXAMINATION DECEMBER 2023**
(First Semester)

Branch - **STATISTICS**

**REAL ANALYSIS AND LINEAR ALGEBRA**

Time: Three Hours

Maximum: 75 Marks

**SECTION-A (10 Marks)**

Answer **ALL** questions

**ALL** questions carry **EQUAL** marks (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | If both right hand limit and left hand limit exist but are not equal in value then the limit of f as x→a<br>a) Exist b) does not exist<br>c) converges d) diverges | K1 | CO1 |
| | 2 | Find c of the mean value theorem if f(x) = x(x-1)(x-2); a=0 b=1/2<br>a) $\frac{6-\sqrt{21}}{6}$ b) $\frac{6}{26}$ c) $\frac{6\sqrt{21}}{6}$ d) $\frac{\sqrt{21}}{6}$ | K2 | CO1 |
| 2 | 3 | If a sequence $\{s_n\}$ has a limit 1 then $\{s_n\}$ converges to 1 is<br>a) Divergent sequence b)convergent sequence<br>c) Monotonic sequence d) Uniform sequence | K1 | CO2 |
| | 4 | The series $\Sigma U_n$ is convergent if limit $(U_n)^{1/n}$ is<br>a) less than zero b) equal to one<br>c) greater than 1 d) less than one | K2 | CO2 |
| 3 | 5 | The lower Riemann integral of f over [a,b] denoted by (R ) $\int_{-a}^{b} f$ is given by<br>a) (R ) $\int_{-a}^{b} f$ = sup {L(f,P )\|P a partition of [a,b]}<br>b) (R ) $\int_{-a}^{b} f$ = inf {U(f,P )\|P a partition of [a,b]}<br>c) (R ) $\int_{-a}^{b} f$ = sup {U(f,P )\|P a partition of [a,b]}<br>d) (R ) $\int_{-a}^{b} f$ = inf {L(f,P )\|P a partition of [a,b]} | K1 | CO3 |
| | 6 | If $f_1(x) \leq f_2(x)$ on [a,b] then<br>a) $\int_a^b f_1(x)\, d\alpha \leq \int_a^b f_2(x)\, d\alpha$<br>b) $\int_a^b f_1(x)\, d\alpha = \int_a^b f_2(x)\, d\alpha$<br>c) $\int_a^b f_1(x)\, d\alpha \geq \int_a^b f_2(x)\, d\alpha$<br>d) $\int_a^b f_1(x)\, d\alpha \neq \int_a^b f_2(x)\, d\alpha$ | K2 | CO3 |
| 4 | 7 | Let V be a vector space and W be a subspace of V then<br>a) U + V = V + U , $\forall\, U, V \in W$<br>b) $k$ U $\in W, \forall\, u \in w, k$ is scalar<br>c) m(nu) = mn (u) , $\forall\, u \in W, m \,\&\, n$ *are scalar*<br>d) All of these | K1 | CO4 |
| | 8 | Any subset of a linearly independent set is<br>a) Linearly dependent b) Linearly independent<br>c) Space d)basis | K2 | CO4 |
| 5 | 9 | The characteristic roots of the matrix $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$ is<br>a) 2,2 b) 3,2 c) -2,2 d) -3,3 | K1 | CO5 |
| | 10 | The nature of quadratic form $X_1^2+2X_2^2$ is<br>a) Negative definite b) Positive definite<br>c) negative semi definite d) Positive semi definite | K2 | CO5 |

**Cont...**

## SECTION - B (35 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Prove that if f is continuous in the closed interval [a,b] then f(x) is bounded in [a,b]. | K2 | CO1 |
| | | (OR) | | |
| | 11.b. | Let I = $]a, b[$ and let f,g be two functions on I into R then Prove that $(f \pm g)' = f' \pm g'$ | | |
| 2 | 12.a. | Using the definition of limit of a sequence show that the sequence $\{s_n\}$ has the limit 3 where $s_n = 3n/(n+5n^{1/2})$. | K3 | CO2 |
| | | (OR) | | |
| | 12.b. | State and Prove Dirichlet's test. | | |
| 3 | 13.a. | If f is continuous on [a,b] then prove that f is R-integrable on [a,b]. | K2 | CO3 |
| | | (OR) | | |
| | 13.b. | State and Prove second Mean Value Theorem. | | |
| 4 | 14.a. | Prove that intersection of two subspaces of a vector space is a subspace. | K5 | CO4 |
| | | (OR) | | |
| | 14.b. | Let V be the vector space of polynomials with inner product given by $< f, g >= \int_0^1 f(t)g(t)\,dt$. Let $f(t) = t+2$ and $g(t) = t^2-2t-3$ find (i) $< f, g >$ and (ii) $\| f \|$ | | |
| 5 | 15.a. | State and Prove Cayley Hamilton Theorem. | K6 | CO5 |
| | | (OR) | | |
| | 15.b. | Explain canonical representation of a quadratic form. | | |

## SECTION -C (30 Marks)

Answer **ANY THREE** questions

**ALL** questions carry **EQUAL** Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | State and Prove Taylor's theorem. | K2 | CO1 |
| 2 | 17 | Find the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots \frac{1}{n^p} + \ldots$ is convergent if $p > 1$, divergent if $p \leq 1$ | K1 | CO2 |
| 3 | 18 | State and prove Fundamental theorem of Integral Calculus. | K2 | CO3 |
| 4 | 19 | Prove that every finite dimensional inner product space has an orthogonal basis. | K5 | CO4 |
| 5 | 20 | Determine the Characteristics Roots and Characteristic Vector of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$ | K5 | CO5 |

Z-Z-Z END