22STU204 / 19STU04

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Second Semester)

Branch – **STATISTICS**

PROBABILITY THEORY / PROBABILITY & DISTRIBUTIONS-I

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The probability of all possible outcomes of a random experiment is always equal to
   (i) infinity (ii) zero
   (iii) one (iv) none of the above
2. Two random variables are said to be independent if
   (i) E (XY) = 1 (ii) E (XY) = E(X) E(Y)
   (iii) E (XY) = 0 (iv) E(XY) = any constant value
3. If an event has occurred and it is known that P(B) = 1, the conditional probability P(A|B) is equal
   (i) P(A) (ii) P(B)
   (iii) Zero (iv) One
4. The moment generating function of Bernoulli distribution is
   (i) $(q+pe^t)^n$ (ii) $(q+pe^t)^{-n}$
   (iii) $(q+pe^t)$ (iv) $(q+pe^{-t})$
5. If E(Y|X) is the conditional expectation of Y given X=x , then
   (i) E(XY) = E(X) E(Y|X) (ii) E(XY) = E(Y) E(Y|X)
   (iii) E(XY) = X E(Y|X) (iv) E(XY) = E[X E(Y|X)]

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Define probability and write the axioms of probability.

   OR

   b) Three coins are tossed simultaneously, find the probability of getting
   (i) atleast one head (ii) Exactly two heads

7. a) Explain random variable and sketch the properties of distribution function.

   OR

   b) A random variable X has probability function as follows:

Values of X :	-1	0	1
Probability :	0.2	0.3	0.5

   Evaluate (i) E(3X+1) (ii) $E(X^2)$

8. a) If the distribution of the random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{4}[1 + xy(x^2 - y^2)] & for\ |x| \le 1, |y| \le 1 \\ 0 & otherwise \end{cases}$$

   Show that X and Y are not independent.

   OR

   b) Define (i) Marginal Probability Distribution
   (ii) Conditional Probability Distribution

Cont...

9. a) Summarize about moment generating function and characteristic function.

OR

b) A die is thrown repeatedly until either 2 or 3 is obtained. Show that the Probability Generating Function of the number of throws required is $\frac{s}{3-2s}$.

10. a) Compare conditional expectation and conditional variance.

OR

b) Let f(x,y) = 8xy, 0<x<y<1; f(x,y) =0 elsewhere.
Find E (Y|X=x).

SECTION -C (30 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) State and prove multiplication theorem of probability

OR

b) Suppose one person is selected at random from a group of 100 persons known to confirm the following patterns of political views.

	Communist	Socialist	Democrat	Total
Men	15	25	10	50
Women	20	15	15	50
Total	35	40	25	100

What is the probability that the man selected is a communist?

12. a) Describe (i) Discrete random variable
(ii) Continuous random variable

OR

b) Find the mean and variance of the probability function

$$p(x) = \begin{cases} \frac{2}{3}\left(\frac{1}{3}\right)^{x-1} & for\, x = 1,2,3 \\ 0 & otherwise \end{cases}$$

13. a) Given the pdf of a continuous random variable X as follows

$$f(x) = \begin{cases} kx(1-x) & for\, x = 1,2,3 \\ 0 & otherwise \end{cases}$$

Find k and the cumulative distribution function.

OR

b) The joint density function of a bivariate distribution is given by

$$f(x,y) = 4xy\, e^{-(x^2+y^2)}, \quad x \geq 0, y \geq 0$$

Find the marginal and conditional probability density functions. Are x and y independent?

14. a) State and prove Tchebychev's inequality.

OR

b) The probability density function of the random variable X follows the probability law:

$$p(x) = \frac{1}{2\theta}\exp\left(-\frac{|x-\theta|}{\theta}\right), -\infty < x < \infty$$

Find M.G.F of X. Hence find E(X) and V(X).

15. a) Given two variates X_1 and X_2 with joint density function f (x_1, x_2), prove that the conditional mean X_2 (given X_1) coincides with (unconditional) mean only if X_1 and X_2 are independent.

OR

b) Given $f(x,y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0, y \geq 0 \\ 0 & otherwise \end{cases}$

Find E(XY) and E(Y|X). Show that E(Y) does not exist.

Z-Z-Z END