

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Third Semester)

Branch – NUTRITION FOOD SERVICE MANAGEMENT & DIETETICS

MATHEMATICS

Time: Three Hours

Maximum: 50 Marks

SECTION-A (5 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(5 x 1 = 5)

1. If $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{pmatrix}$, then the eigen values of A^2 are ____.

- a) $-1, 3, -2$ b) $-1, \frac{1}{3}, -\frac{1}{2}$ c) $-1, 27, -8$ d) $1, 9, 4$.

2. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$ then the value of

$(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$ is ____.

- a) r b) $-r$ c) r^2 d) $-r^2$.

3. When Gauss elimination method is used to solve a system of linear equations, the coefficient matrix is reduced to a ____ matrix.

- a) diagonal b) identity
c) lower triangular d) upper triangular

4. Lagrange's interpolation formula is applied only when the independent variable advances by ____.

- a) unequal intervals b) equal intervals
c) equal and unequal intervals d) none

5. The value of $\frac{1}{h^2} [\Delta^2 y_0 + \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 + \dots]$ is ____.

- a) $\left(\frac{d^2 y}{dx^2}\right)_{x=x_n}$ b) $\left(\frac{d^3 y}{dx^3}\right)_{x=x_n}$ c) $\left(\frac{d^3 y}{dx^3}\right)_{x=x_0}$ d) $\left(\frac{d^2 y}{dx^2}\right)_{x=x_0}$

SECTION - B (15 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks

(5 x 3 = 15)

6.a) Find the sum and product of the eigen values of the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$.

Or

b) Find the eigen values and eigen vectors of the matrix: $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$.

Cont...

7.a) Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that $1 - \sqrt{5}$ is a root.

Or

b) Show that if the roots of the equation $x^3 + px^2 + qx + r = 0$ are in Arithmetic

Progression then $2p^3 - 9pq + 27r = 0$.

8.a) Solve the following system of equations by Gauss elimination method:

$$5x - y = 9, \quad -x + 5y - z = 4, \quad -y + 5z = -6.$$

Or

b) Tabulate three iterate values for the following system of equations by Gauss Jacobi:

$$30x - 2y + 3z = 75, \quad x + 17y - 2z = 48, \quad x + y + 9z = 15.$$

9.a) Find the quadratic polynomial passing through the points (0, 0), (1, 1) and (3, 0) by using Lagrange's method.

Or

b) Find $f(6)$ by Newton's divided difference method from the following data:

x	1	2	7	8
f(x)	1	5	5	4

10.a) From the following table to find dy/dx at $x = 1$:

X	1	2	3	4
Y	1	8	27	64

Or

b) Using Trapezoidal rule find $\int_0^1 f(x) dx$ from the following data:

x	0	0.25	0.5	0.75	1
f(x)	1	0.9394	0.7788	0.5698	0.3678

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 x 6 = 30)

11.a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$.

Or

b) Apply the Cayley-Hamilton theorem, to obtain inverse of the matrix $A = \begin{pmatrix} -2 & -3 \\ 6 & 7 \end{pmatrix}$.

12.a) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ find the value of

i) $\sum (\beta + \gamma + \delta)^2$ ii) $\sum \frac{1}{\alpha^2}$.

Or

b) Increase the roots of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ by 2 and hence solve the equation.

Cont...

13.a) Find the solution to the following equations by Gauss-Jordan method:

$$2x + y + 4z = 12, 8x - 3y + 2z = 20, 4x + 11y - z = 33.$$

Or

b) Solve the following system of equations by Gauss Seidel method (correct up to three decimal places): $27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$.

14.a) From the following data, find the value of y when $x = 43$ by using Newton's forward interpolation formula:

X	40	50	60	70	80	90
Y	184	204	226	250	276	304

Or

b) Find the cubic polynomial which approximate the following data by Lagrange's interpolation formula:

x	-2	-1	2	3
f(x)	-12	-8	3	5

15.a) From the following table of values of x and y , find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

Or

b) Evaluate $\int_0^\pi \sin x \, dx$ using Simpson's $\frac{1}{3}$ rd rule by taking ten equal parts.

Z-Z-Z END