PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 An equation is said to be linear if it is of first degree in the _____
(i) independent variable (ii) dependent variable
(iii) derivatives (iv) lower derivatives

2 The complete integral of $p^2 + q^2 = 1$
(i) $z = ax + \sqrt{a^2 - 1}\ y + c$ (ii) $z = ax + \sqrt{1 + a^2}\ y + c$
(iii) $z = ax + \sqrt{1 - a^2}\ y + c$ (iv) $z = ax - \sqrt{1 - a^2}\ y + c$

3 If all the derivatives in a partial differential equation is of same order then it is called as _____ equation
(i) linear (ii) non linear
(iii) linear homogenous (iv) linear non homogeneous

4 The positive number P is called period of $f(x)$, if
(i) $f(x + p) = f(x)$ (ii) $f(x + p) = f(p)$
(iii) $f(x - p) = f(p)$ (iv) $f(p - x) = f(p)$

5 The eigen values of the vibrating string is _____
(i) $\lambda_n = cn\pi L$ (ii) $\lambda_n = -cn\pi L$
(iii) $\lambda_n = cn/\pi L$ (iv) $\lambda_n = cn\pi/L$

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a) Construct the partial differential equation by eliminating a,b and c from $z = a(x + y) + b(x - y) + abt + c$

OR

b) Solve $y^2p - xyq = x(z - 2y)$

7. a) Solve p + q = pq

OR

b) Evaluate the complete integral of $9(p^2z + q^2) = 4$

8. a) Solve 25r – 40s +16t = 0

OR

b) Solve $(D^2 - 2DD' + D'^2)z = 12xy$

Cont...

9. a) Evaluate the power series of the function

$$f(x) = \begin{cases} -k & if \quad -2 < x < 0 \\ k & if \quad 0 < x < 2 \end{cases} \quad p = 2L = 4$$

OR

b) Evaluate the Fourier series of the function

$f(x) = x + \pi \ \ if \ -\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

10. a) Calculate the ttemperature u(x,t) in a laterally insulated copper bar 80 cm long if the initial temperature is 100 $\sin(3\pi x/80)^{\circ}C$ and the ends are kept at $0^{\circ}C$. How long it will take for the maximum temperature in the bar to drop to $50^{\circ}C$? (where density = 8.92b/cm^3, specific heat = 0.092cal/$g^{\circ}C$, thermal conductivity = 0.95 $cm sec^{\circ}C$).

OR

b) Calculate the temperature of a bar of length L whose ends are insulated and the initial temperature is

$$f(x) = \begin{cases} x & if \quad 0 < x < \frac{L}{2}, \\ L - x & if \quad L/2 < x < L \end{cases}$$

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a) Construct a partial differential equation by eliminating the arbitrary function φ from $\varphi(x + y + z, x^2 + y^2 + z^2 = 0)$.

OR

b) Solve $z(p - q) = z^2 + (x + y)^2$

12. a) Evaluate the complete integral of $(x^2 + y^2)(p^2 + q^2) = 1$.

OR

b) Evaluate the complete integral of $q^2y^2 = z(z - px)$.

13. a) Solve $(D^3 - 4D^2D' + 4DD'^2)z = 4\sin(2x + y)$.

OR

b) Solve $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$.

14. a) Let $f(x)$ be periodic with period 2π and piecewise continuous in the interval $-\pi \leq x \leq \pi$. Furthermore $f(x)$ have a left hand derivative and a right hand derivative at each point, then prove that the Fourier series $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n cosnx + b_n \sin nx)$ converges.

OR

b) A sinusoidal voltage $Esin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Evaluate the Fourier series of the resulting periodic function.

$$u(t) = \begin{cases} 0 & if \quad -L < t < 0, \\ Esin\omega t & if \quad 0 < t < L \end{cases} \quad p = 2L = \frac{2\pi}{\omega}$$

15 a) Derive the one dimensional wave equation.

OR

b) Determine the solution of $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$, $u(x,t) = 0$ when $u(0,t) = 0$, $u(L,t) = 0$ for all $t \geq 0$ which is replaced by the condition that both ends of the bar are insulated.

Z-Z-Z END