18MCU23

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2023
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATION**

LINEAR ALGEBRA

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. In a vector space, α (v – w) = ____________ .
   (i)αv -αw (ii)$\frac{\alpha v}{\alpha w}$ (iii)α - βw (iv)αv + w

2. If $\dim_F$ v = m then $\dim_F$ Hom (v, v) = _________.
   (i)n (ii)m^2 (iii)–n (iv)m + n

3. An element in A(v) which is not regular is called
   (i)regular (ii)singular (iii)Euler (iv)irreducible

4. A _______ for a subspace H of $\mathbf{R}^n$ is a linearly independent set in H that spans H.
   (i)basis (ii)null set (iii)angle (iv)empty set

5. An nxn matrix with n distinct eigen values is __________.
   (i)diagonalizable (ii)IVP (iii)orthonormal (iv)Jacobis method

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a If V is the internal direct sum of $U_1, \ldots, U_n$, then prove that V is isomorphic to the external direct sum of $U_1, \ldots, U_n$.

OR

   b If $v_1, \ldots, v_n$ are in V then show that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, \ldots, v_{k-1}$.

7. a Prove that A(A(W)) = W.

OR

   b If u, v$\in$V and α, $\beta\in$F then show that
   $$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}(u, u) + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + \beta\bar{\beta}(v, v).$$

8. a If V is finite – dimensional over F, then prove that T$\in$A(V) is invertible if 3 and only if the constant term of the minimal polynomial for T is not 0.

OR

   b If m(s) = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and m(T) = $\begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$ then, find m(ST).

9. a Determine the rank of the matrix A = $\begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$.

OR

Cont...

b If $A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}$ and $C = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$, then find AC and CA.

10. a Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are u and v eigenvectors of A?

OR

b Find the characteristic equation of $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a If $v_1, \ldots, v_n$ is a basis of V over F and if $w_1, \ldots, w_m$ in V are linearly independent over F, then prove that $m \leq n$.

OR

b If V is a vector space over F then show that

(i) $\alpha 0 = 0$ for $\alpha \in F$, (ii) $ov = 0$ for $v \in V$

(iii) $(-\alpha) v = -(\alpha v)$ for $\alpha \in F$, $v \in V$

(iv) If $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.

12. a If V and W are of dimensions m and n, respectively, over F, then prove that Hom (V, W) is of dimension mn over F.

OR

b Let V be a finite – dimensional inner product space; then show that V has an orthonormal set as a basis.

13. a If A is an algebra, with unit element, over F, then show that A is isomorphic to a subalgebra of A(V) for some vector space V over F.

OR

b (i) If V is finite – dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(ii) If $T \in A(V)$ and if $S \in A(V)$ is regular, then show that $r(T) = r(STS^{-1})$.

14. a Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

OR

b Find a basis for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

15. a Determine if the following matrix is diagonalizable, $A = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}$.

OR

b Let $A = \begin{bmatrix} .5 & -.6 \\ .75 & 1.1 \end{bmatrix}$ find the eigenvalues of A, and find a basis for each eigenspace.

Z-Z-Z END